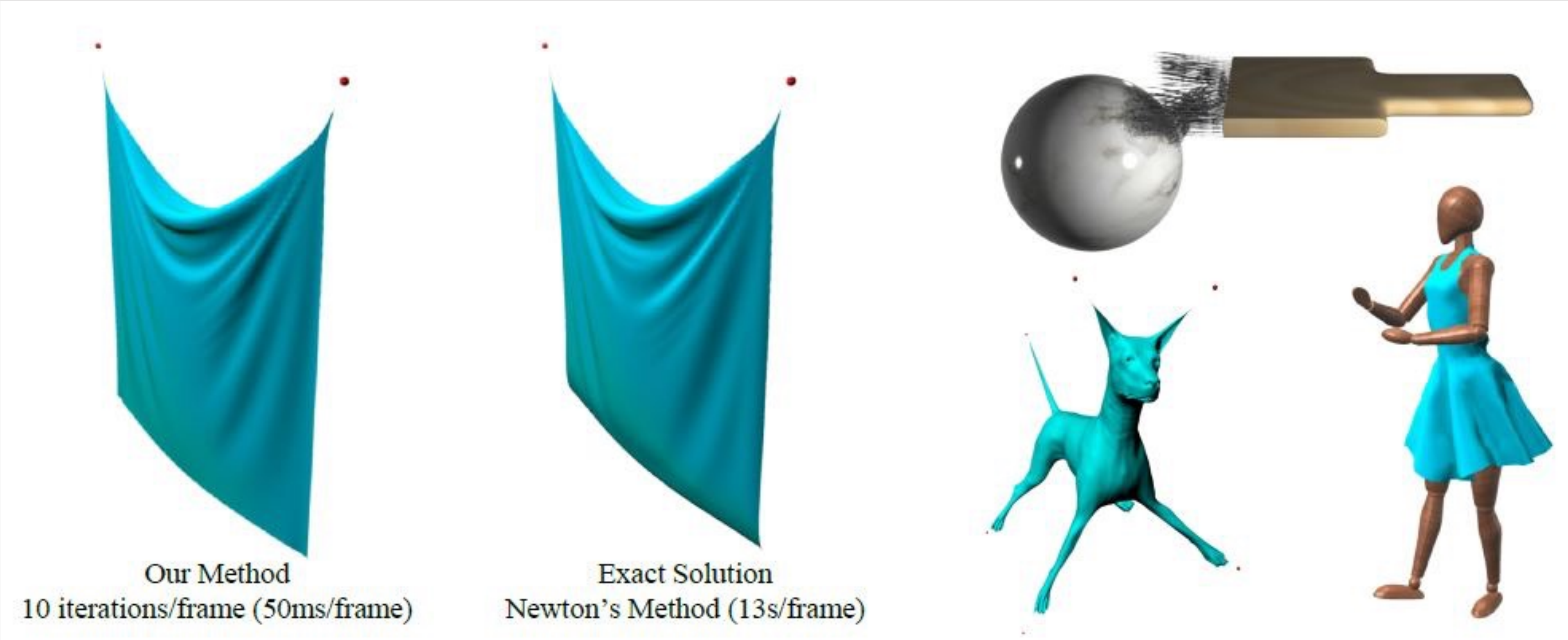
Mass-Spring System

バネ・質点モデル

Widely Used Model for Elastic Objects

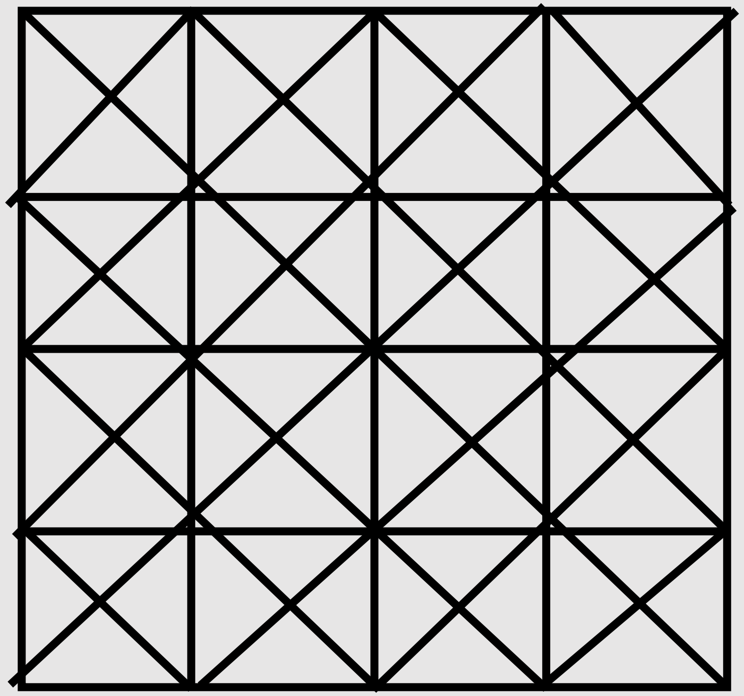


Tiantian Liu, Adam W. Bargteil, James F. O'Brien, Ladislav Kavan. **Fast Simulation of Mass-Spring Systems**. *ACM Transaction on Graphics 32(6) [Proceedings of SIGGRAPH Asia], 2013.*

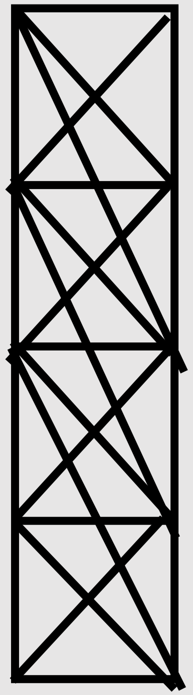
Modeling Cloth, Rod, and Solid

- Heuristic layout of springs to prevent undesirable deformation

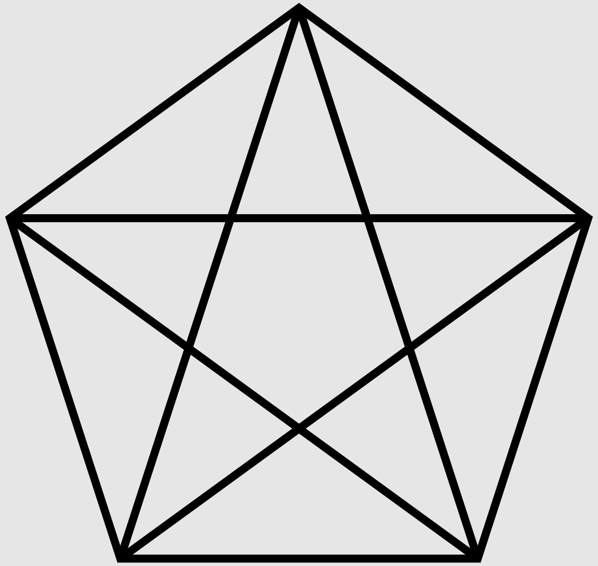
Cloth



Rod

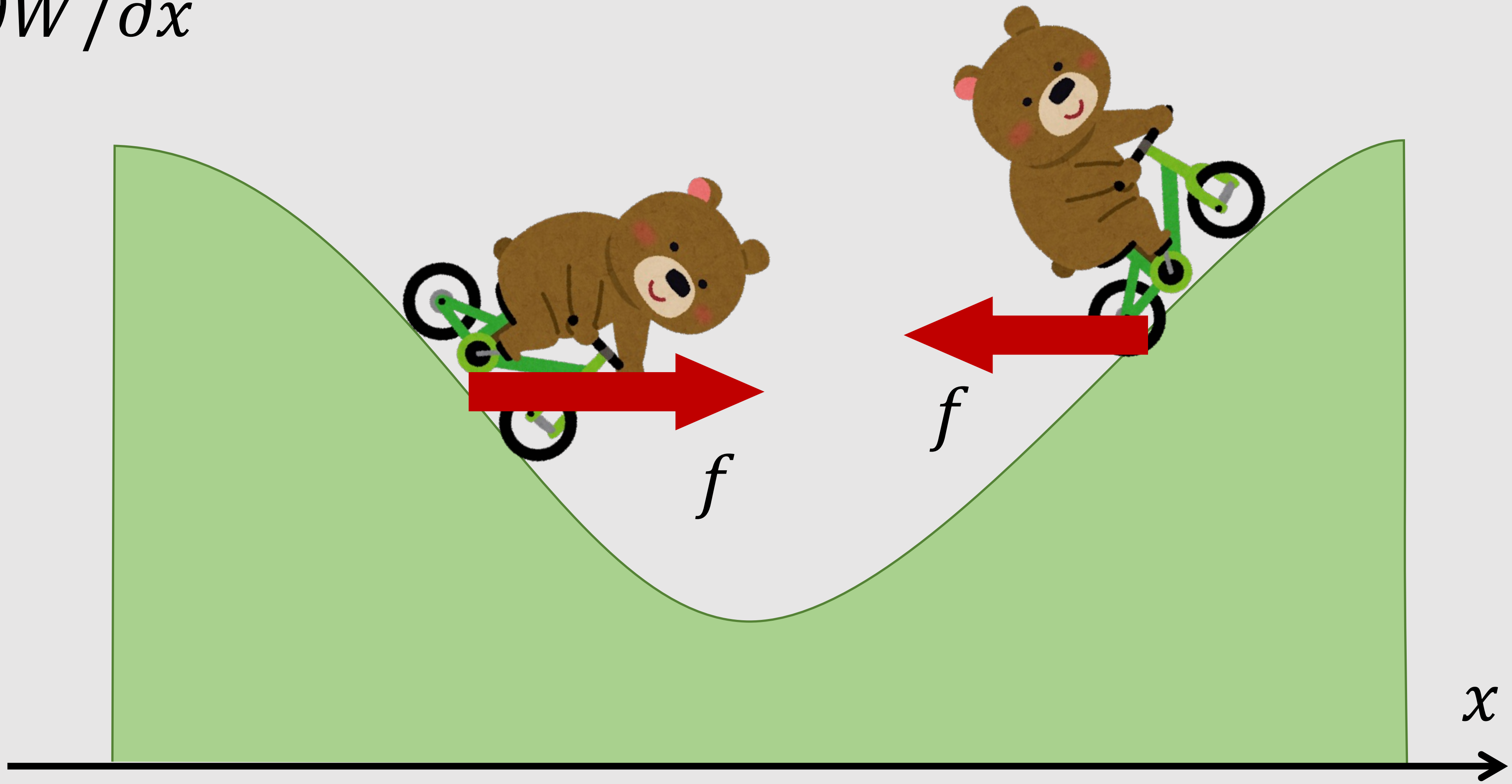


Solid



Potential Energy: Energy Given by Position

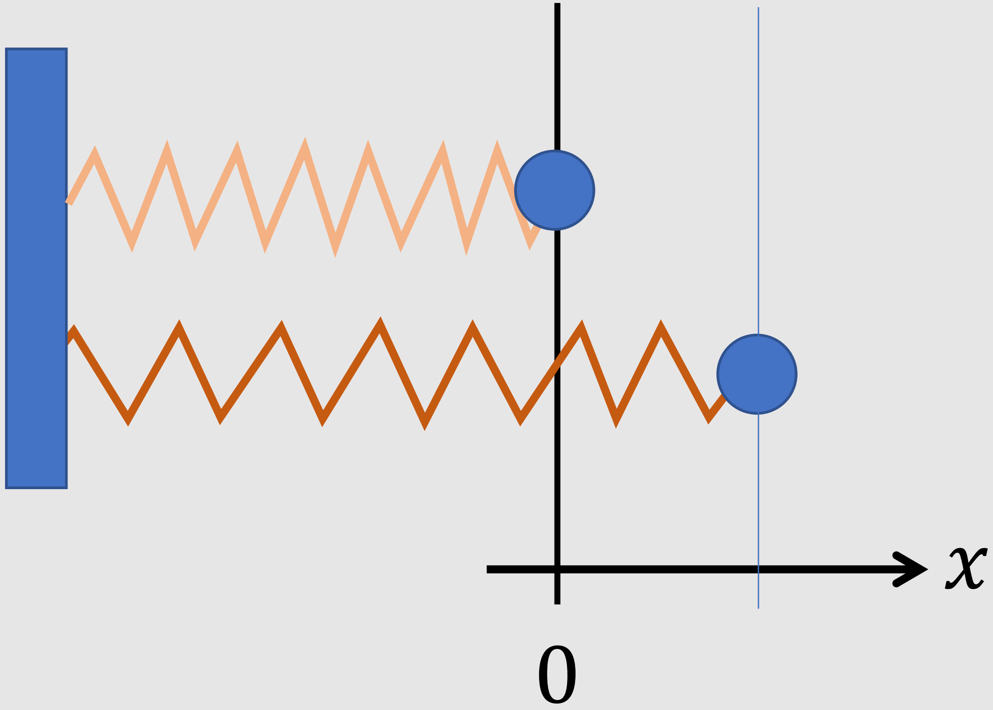
- Gravitational potential energy: $W = -mgh$
- Force: $f = -\partial W / \partial x$



Hooke's Law

- Force changes linearly to the displacement

L: rest length



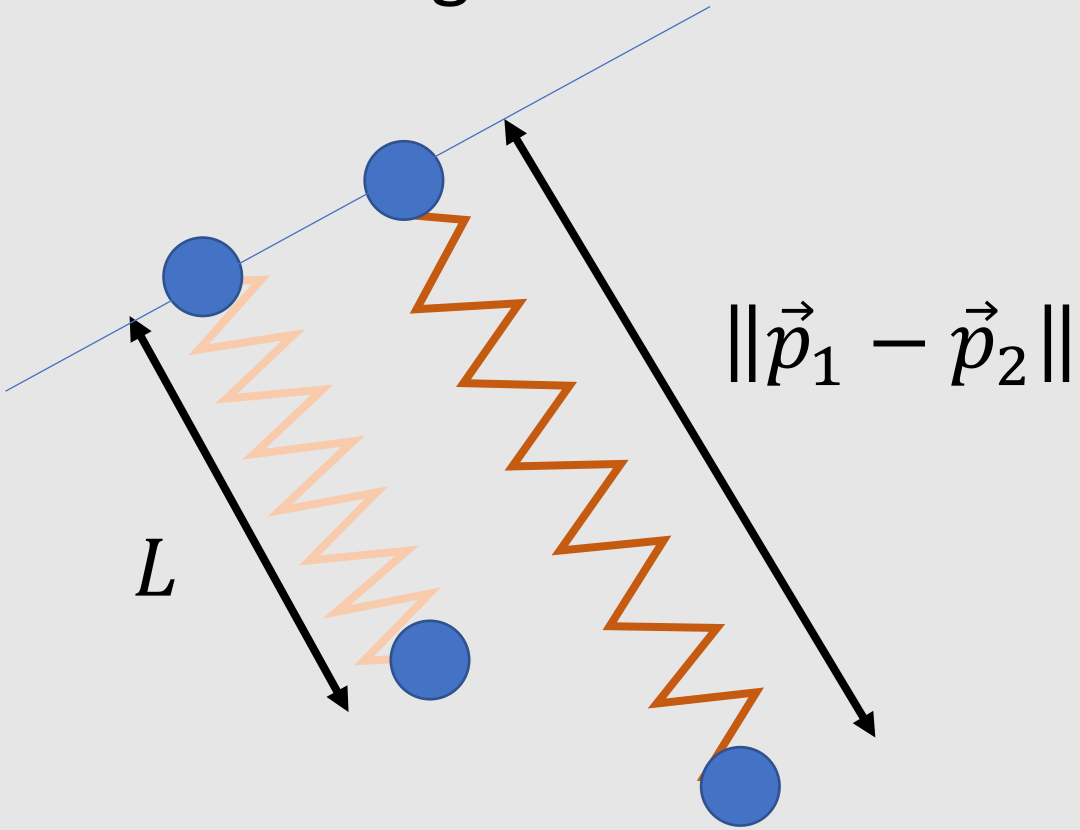
$$f = -kx$$

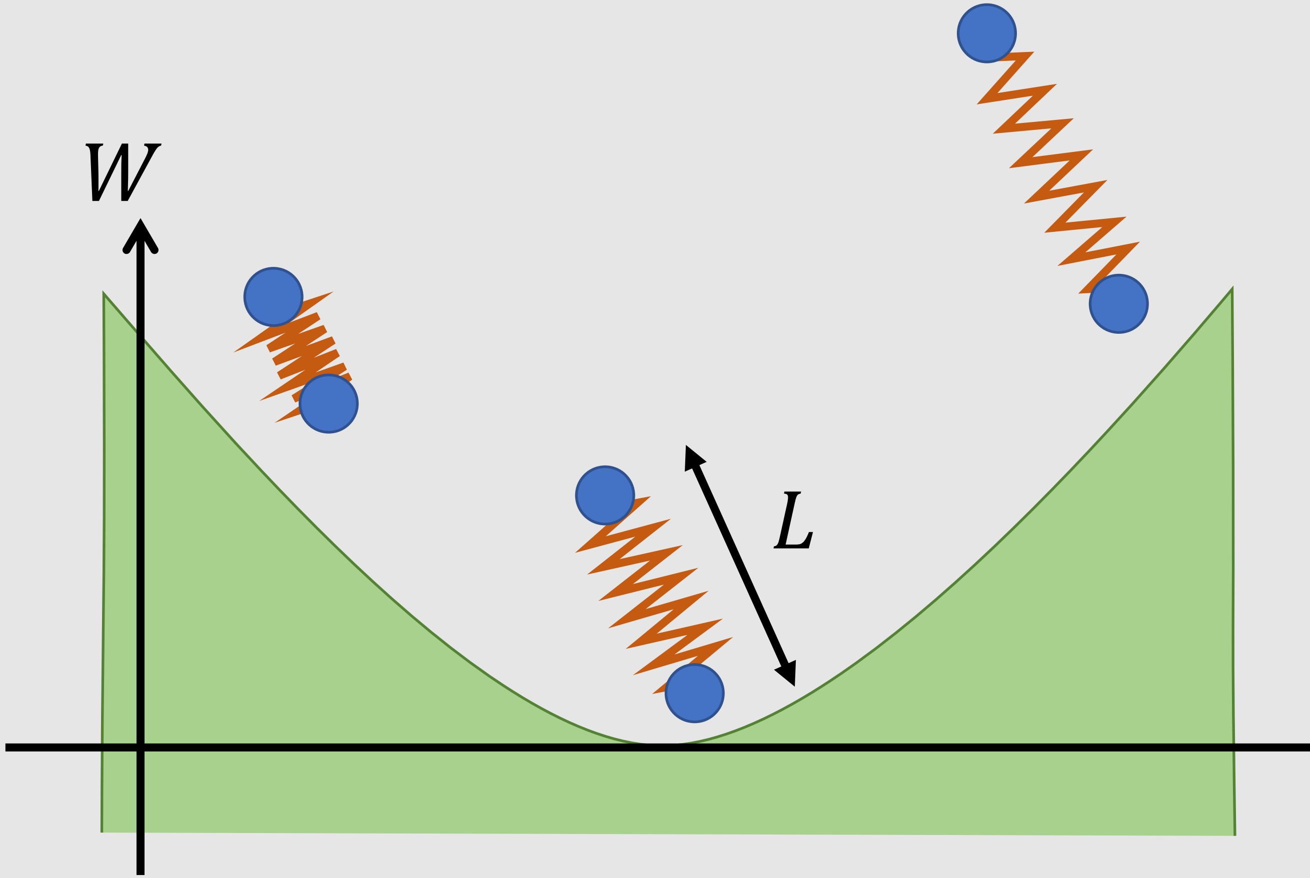
$$W = \int_0^x f\,dx = ?$$

A Spring in 3D

L: rest length

$$W(\vec{p}_1, \vec{p}_2) = \frac{1}{2}k(\|\vec{p}_1 - \vec{p}_2\| - L)^2$$

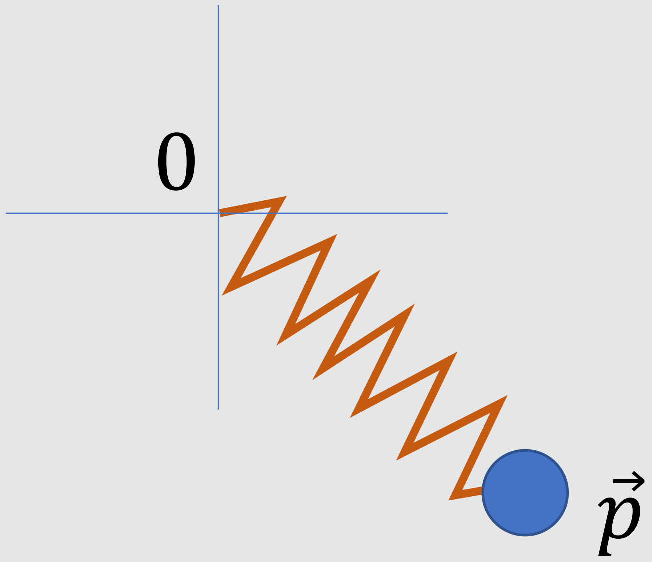




Force of the 3D Spring

- One end is fixed to the origin

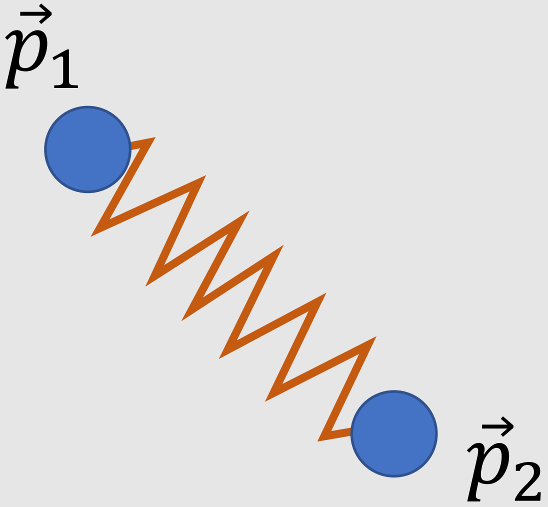
$$W(\vec{p}) = \frac{1}{2}k(||\vec{p}|| - L)^2$$



$$f = \frac{\partial W}{\partial \vec{p}} = ?$$

Force of the 3D Spring

- Both ends are free



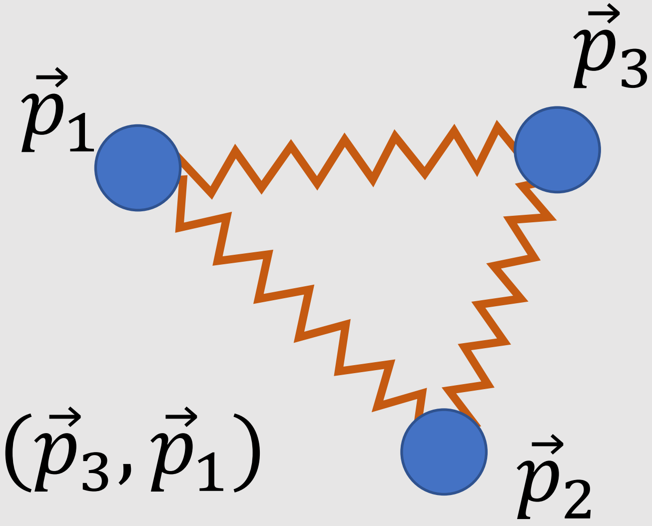
$$W(\vec{p}_1, \vec{p}_2) = \frac{1}{2} k(\|\vec{p}_1 - \vec{p}_2\| - L)^2$$

$$f_1 = \frac{\partial W}{\partial \vec{p}_1} = ?$$

$$f_2 = \frac{\partial W}{\partial \vec{p}_2} = ?$$

check it out!

Three Springs



- Summing up three energy terms

$$W_{total}(\vec{p}_1,\vec{p}_2,\vec{p}_3) = W(\vec{p}_1,\vec{p}_2) + W(\vec{p}_2,\vec{p}_3) + W(\vec{p}_3,\vec{p}_1)$$

$$f_1 = \frac{\partial W_{total}}{\partial \vec{p}_1} = ?$$

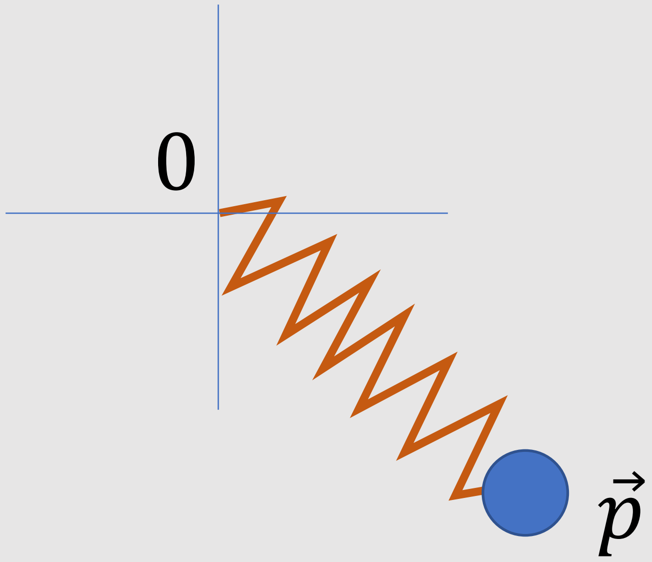
$$f_2 = \frac{\partial W_{total}}{\partial \vec{p}_2} = ?$$

$$f_3 = \frac{\partial W_{total}}{\partial \vec{p}_3} = ?$$

check it out!

Hessian of Elastic Potential Energy

- One end is fixed to the origin



$$W(\vec{p}) = \frac{1}{2}k(||\vec{p}|| - L)^2$$

$$\frac{\partial W}{\partial \vec{p}} = ?$$