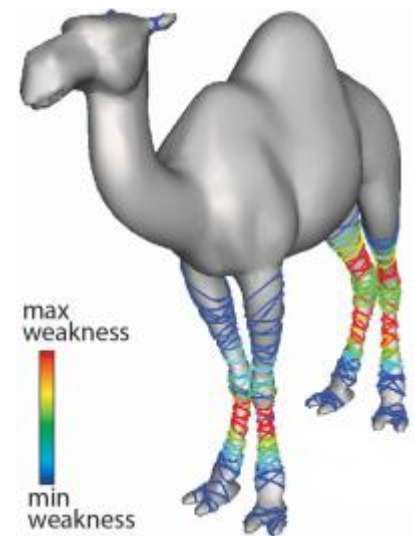


# Cross-sectional Structural Analysis for 3D Printing Optimization

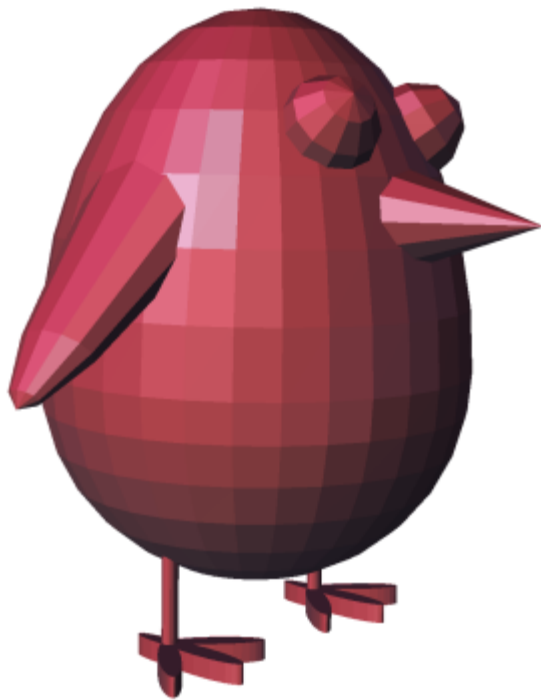
Nobuyuki Umetani   Ryan Schmidt



# Shape Design for 3D Printing is Difficult

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Beginners cannot design structurally sound shape



[source: DBrager14@Thingiverse]

3D printing



(1-2 hours)



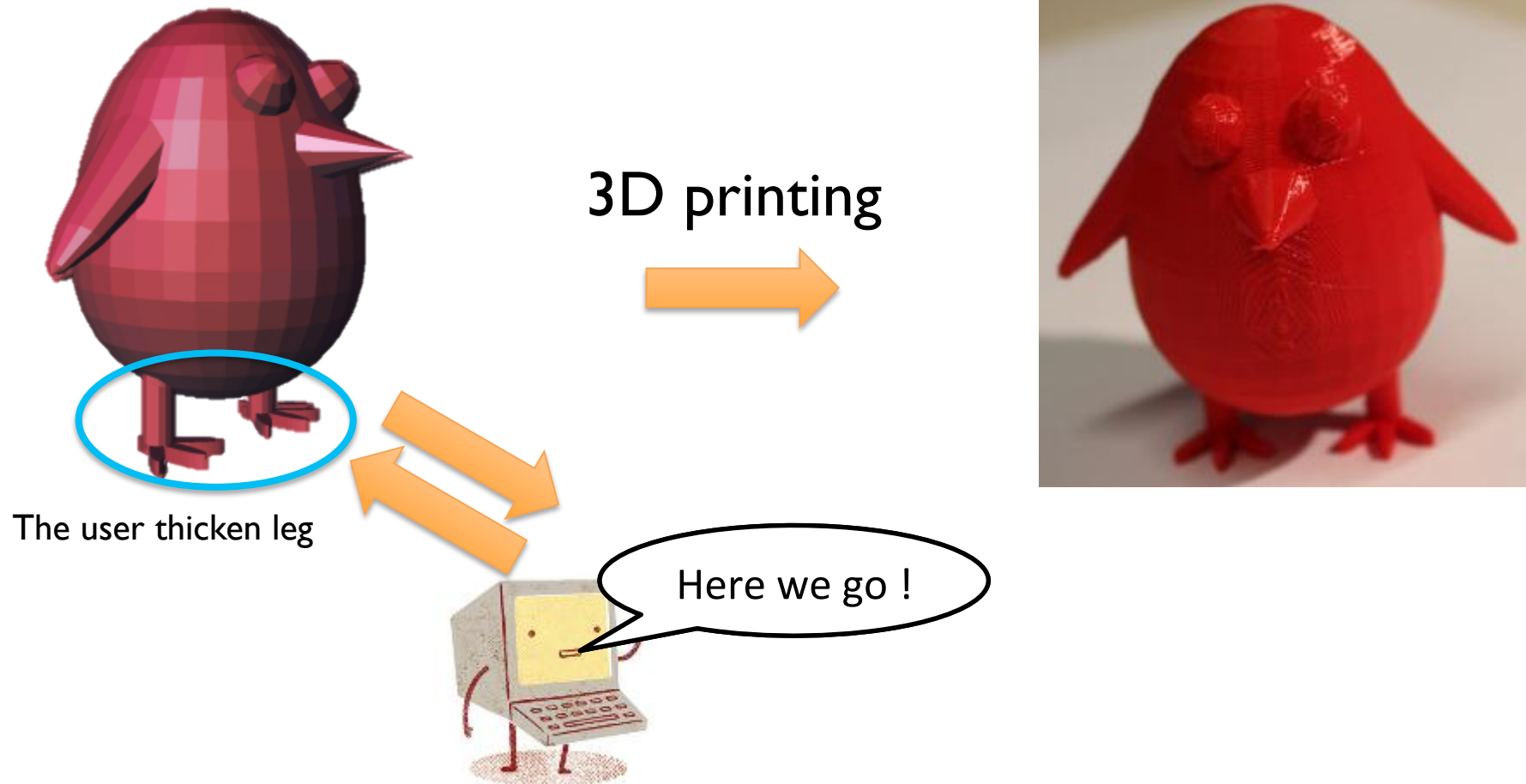
(1-3 weeks)

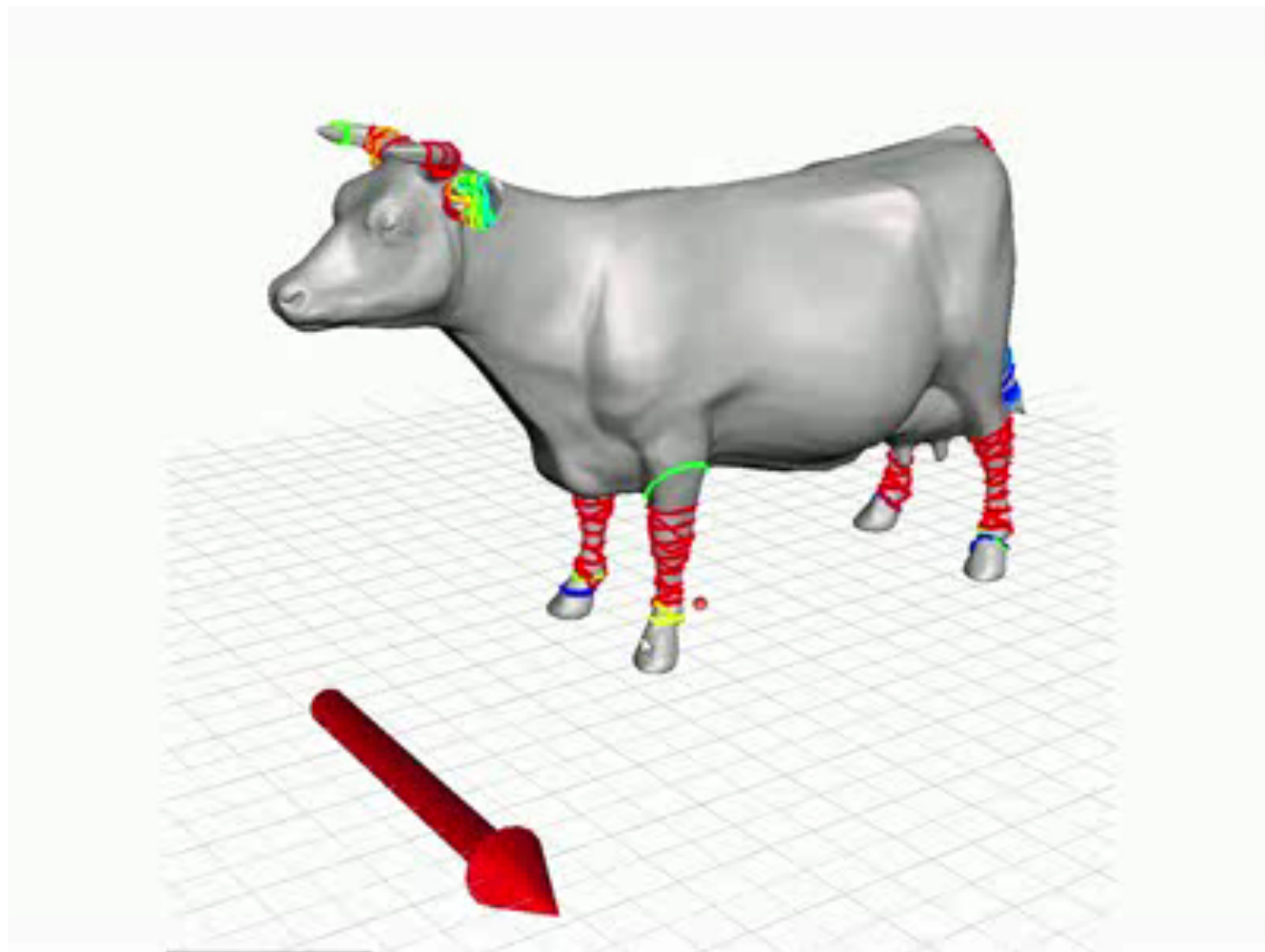


# Goal: Real-time Weakness Detection During Design

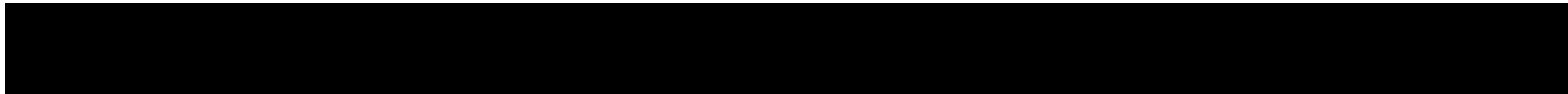
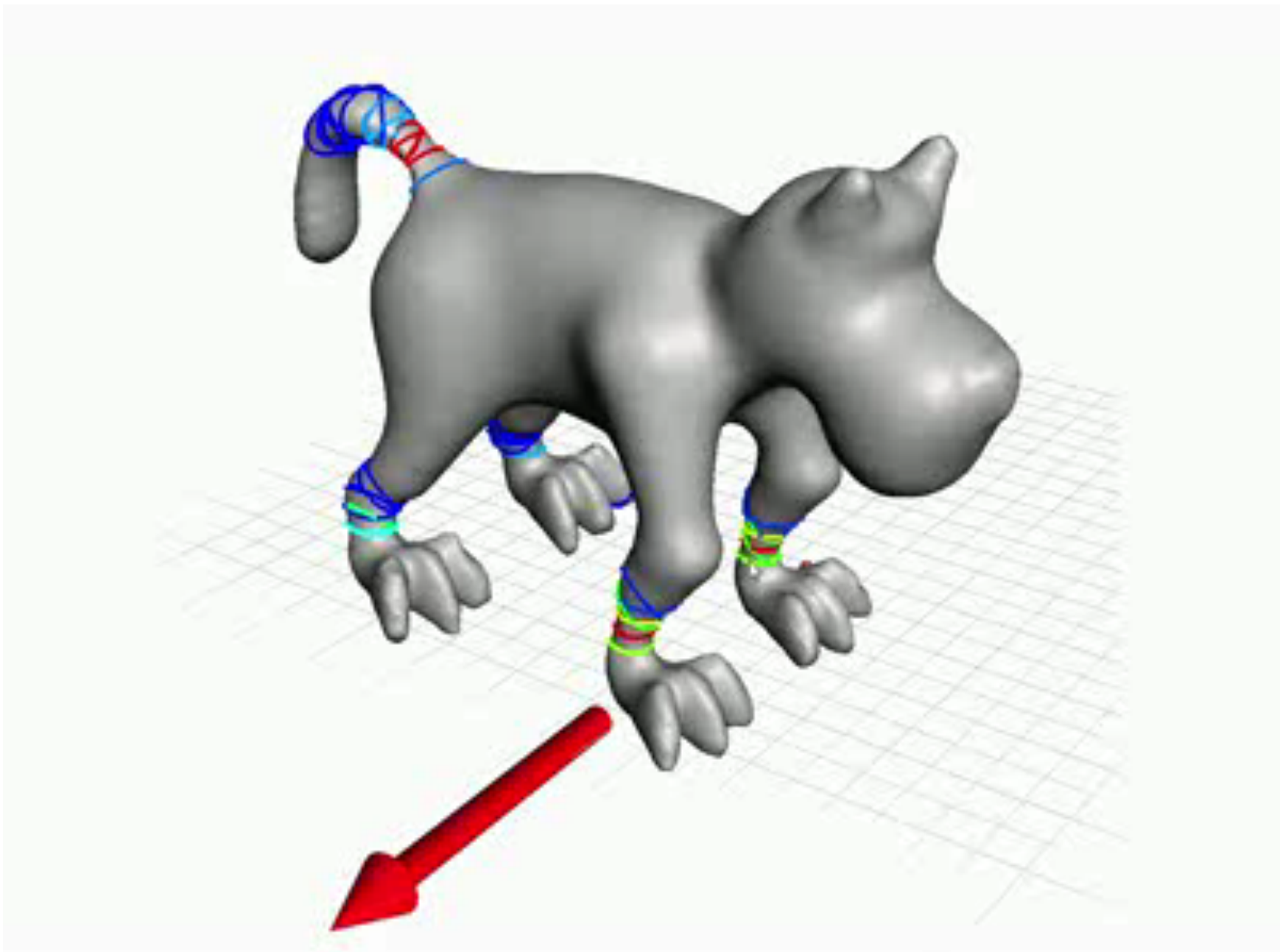
---

Design with interactive trial & error



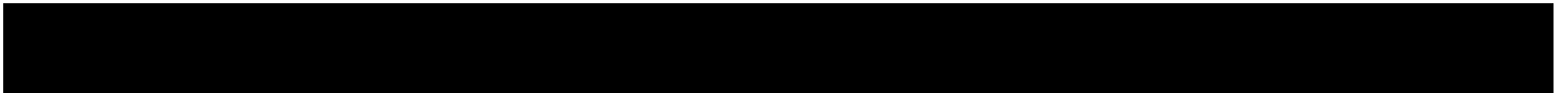






## **Related Works**

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# Related Works: Simulation during Interactive Design

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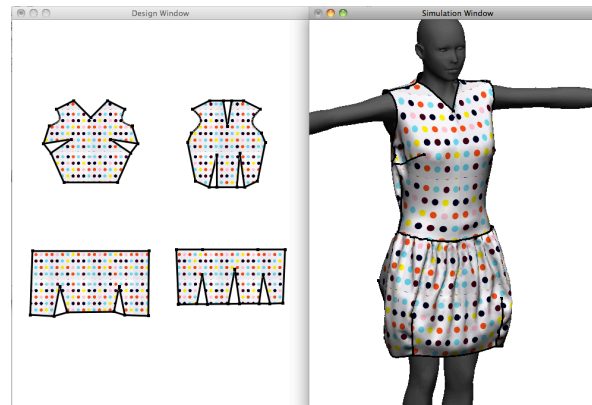
Structural analysis for solid 3D shape had been extremely difficult



[Umetani et al. NIME 2010]



[Furuta et al. Eurographics 2010]



[Umetani et al. Siggraph 2011]

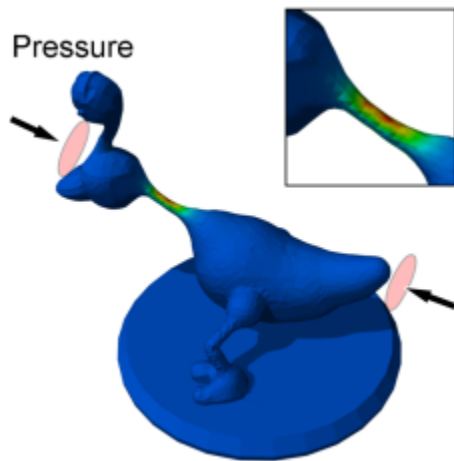


[Umetani et al. Siggraph 2012]

# Related Works: Structural Weakness Detection

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All previous works are based on **Finite Element Method**



[Stava et al. Siggraph 2012]



[Luo et al. Siggraph Asia 2012]



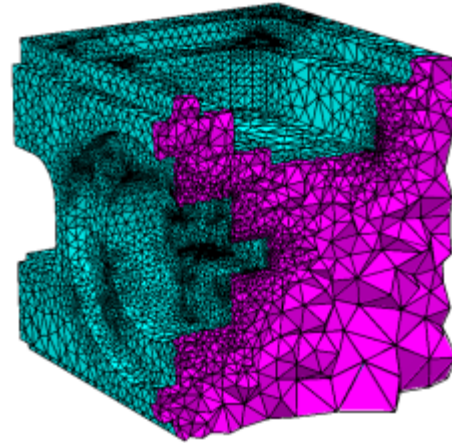
[Zhou et al. Siggraph 2013]

# Drawbacks of Finite Element Method

---

- Slow

- Mesh construction
- Linear solver

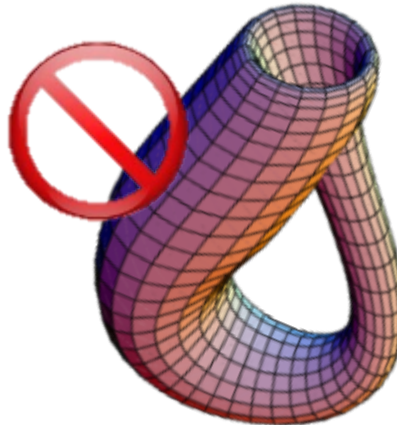
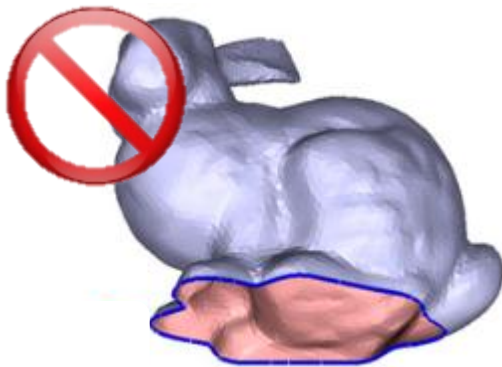


- FEM requires fair mesh

- Without hole & intersection

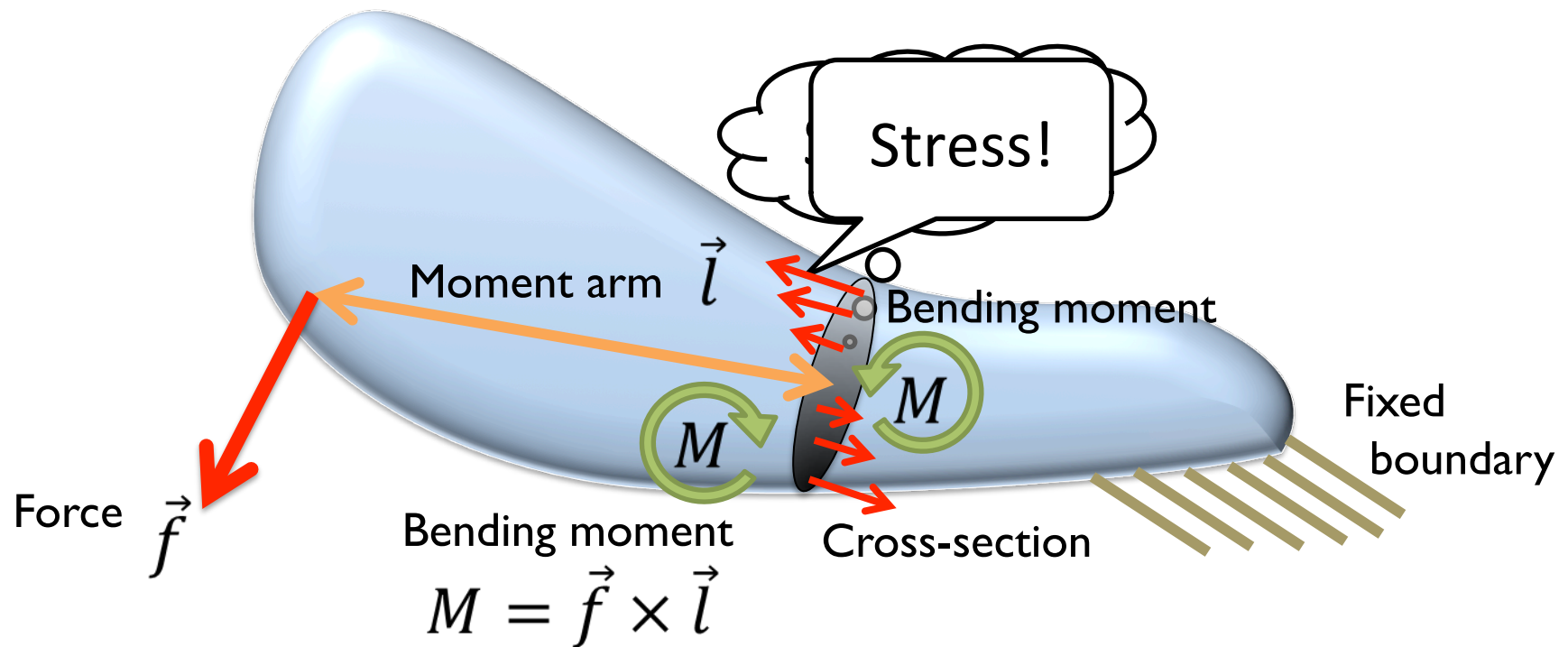
(only less than 25% meshes are closed manifold, 90% are self-intersecting [Gilbert 2013])

- Well-shaped triangle



# Solution: Cross Sectional Structural Analysis

Key idea: bending momentum conservation

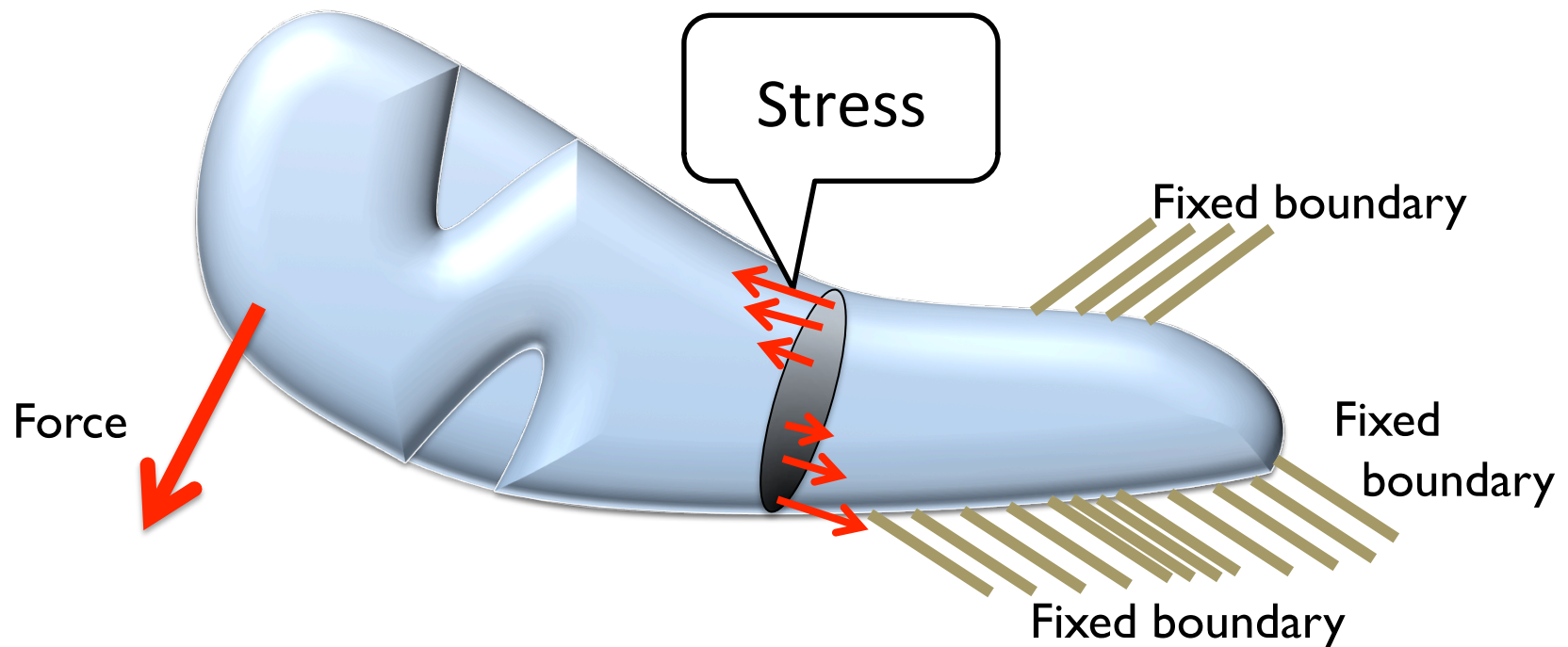


# Problem is simplified very much

---

We don't care {

- Location of fixed boundaries
- Object's in-between 3D shape



# How the structure break ?

---

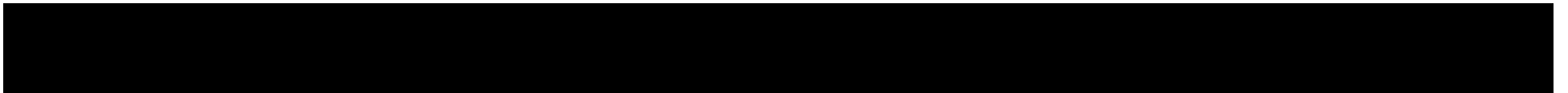
Structure breaks at **slender parts**



Structure breaks by **bending**



How a beam bends?





# History of the Beam Theory

[Galileo 1638]



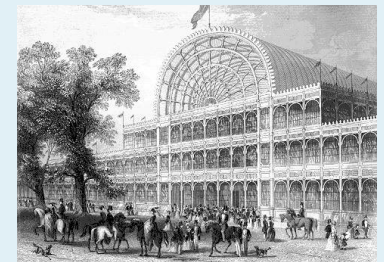
[source: The Stillman Drake Collection]

[Euler 1750]



[source: wikipedia]

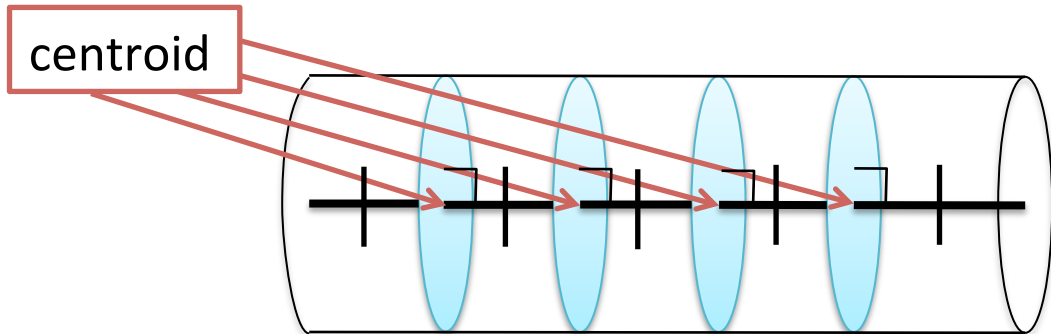
2<sup>nd</sup> industrial revolution



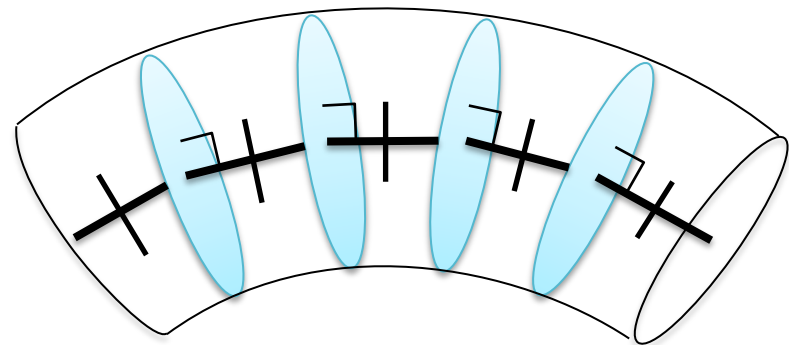
[ source:wikipedia]

# Euler-Bernoulli Assumption

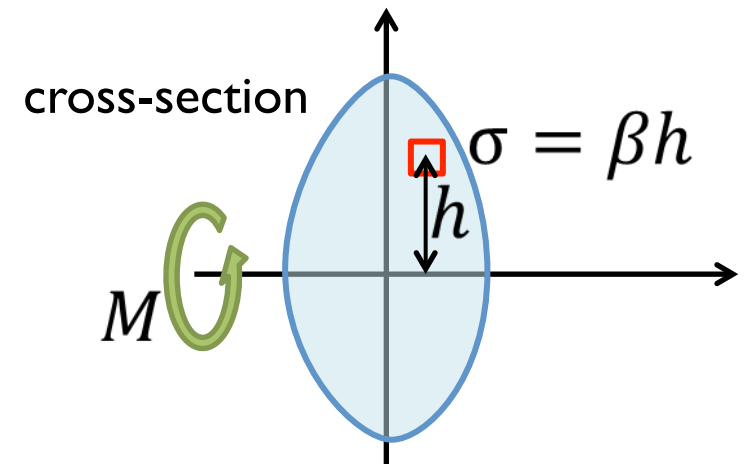
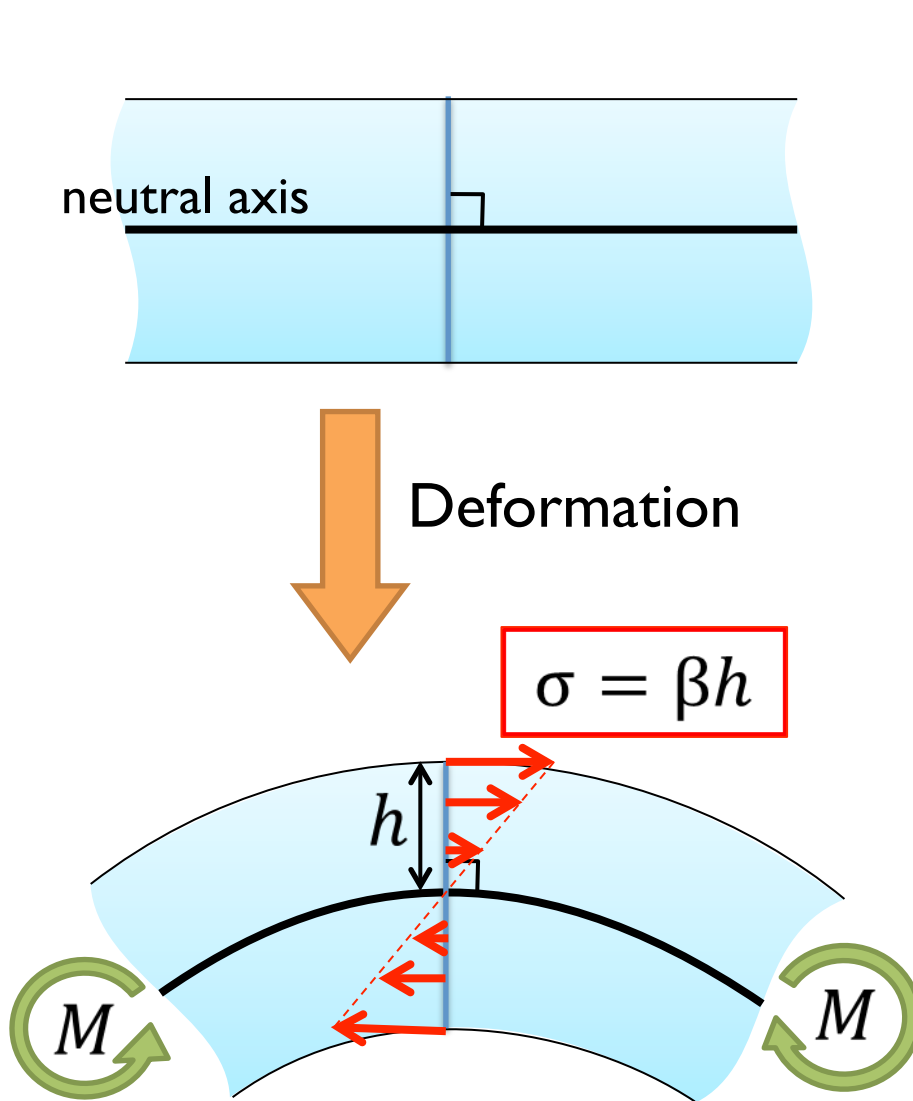
- Neutral axis



- Cross sections do not deform
- Neutral axis does not stretch
- Cross sections remain orthogonal to neutral axis



# Stress on a Cross-Section



$$M = \int_{\Omega} \sigma h d\Omega = \int_{\Omega} \beta h^2 d\Omega$$

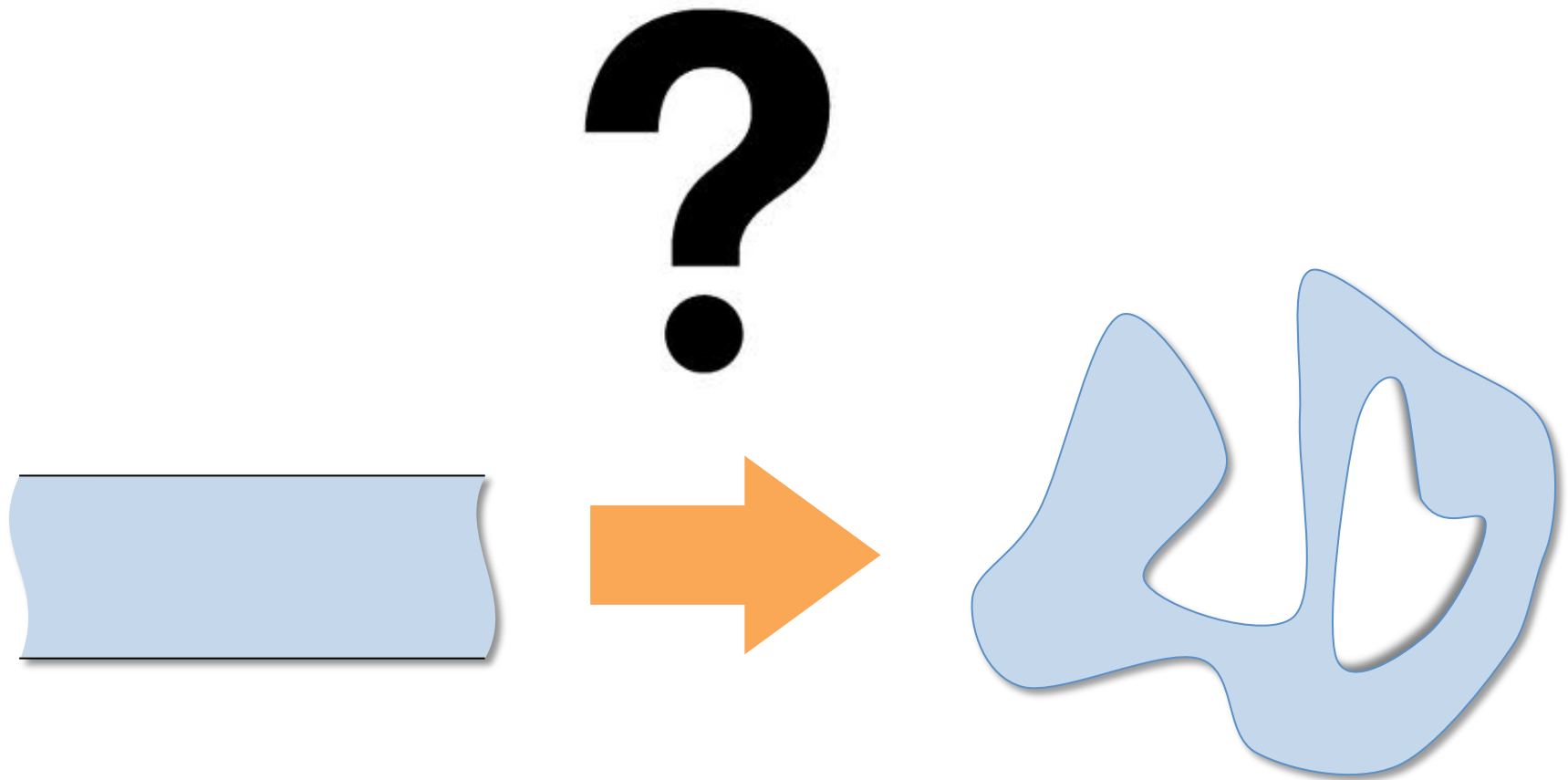
$$\beta = \frac{M}{\int_{\Omega} h^2 d\Omega} \quad \text{Second moment of area}$$

Fracture condition

$$\sigma_{max} = \beta h_{max} > \sigma_{break}$$

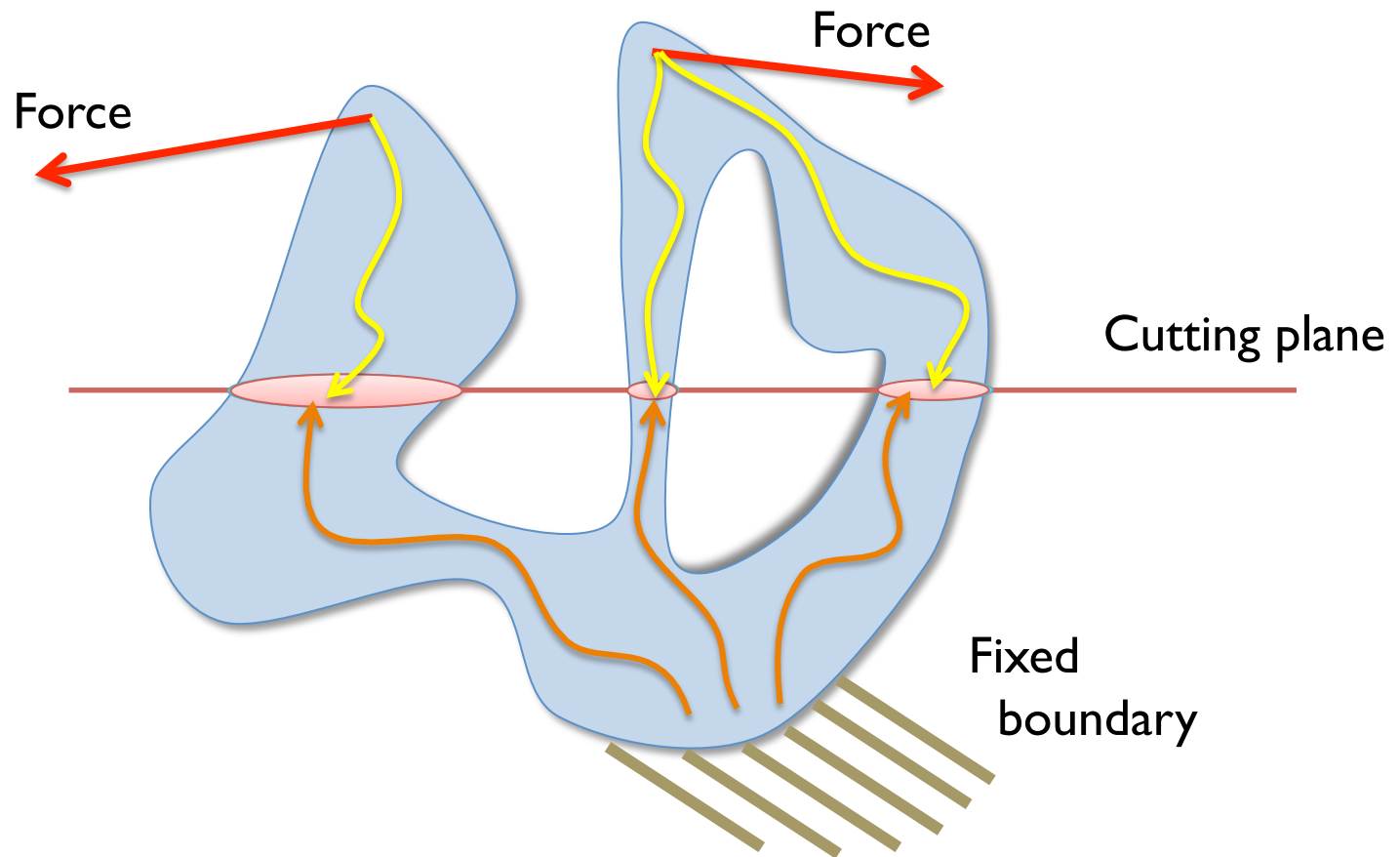
# How we can extend beam theory into 3D shape?

---



# Extension to 3D Shape

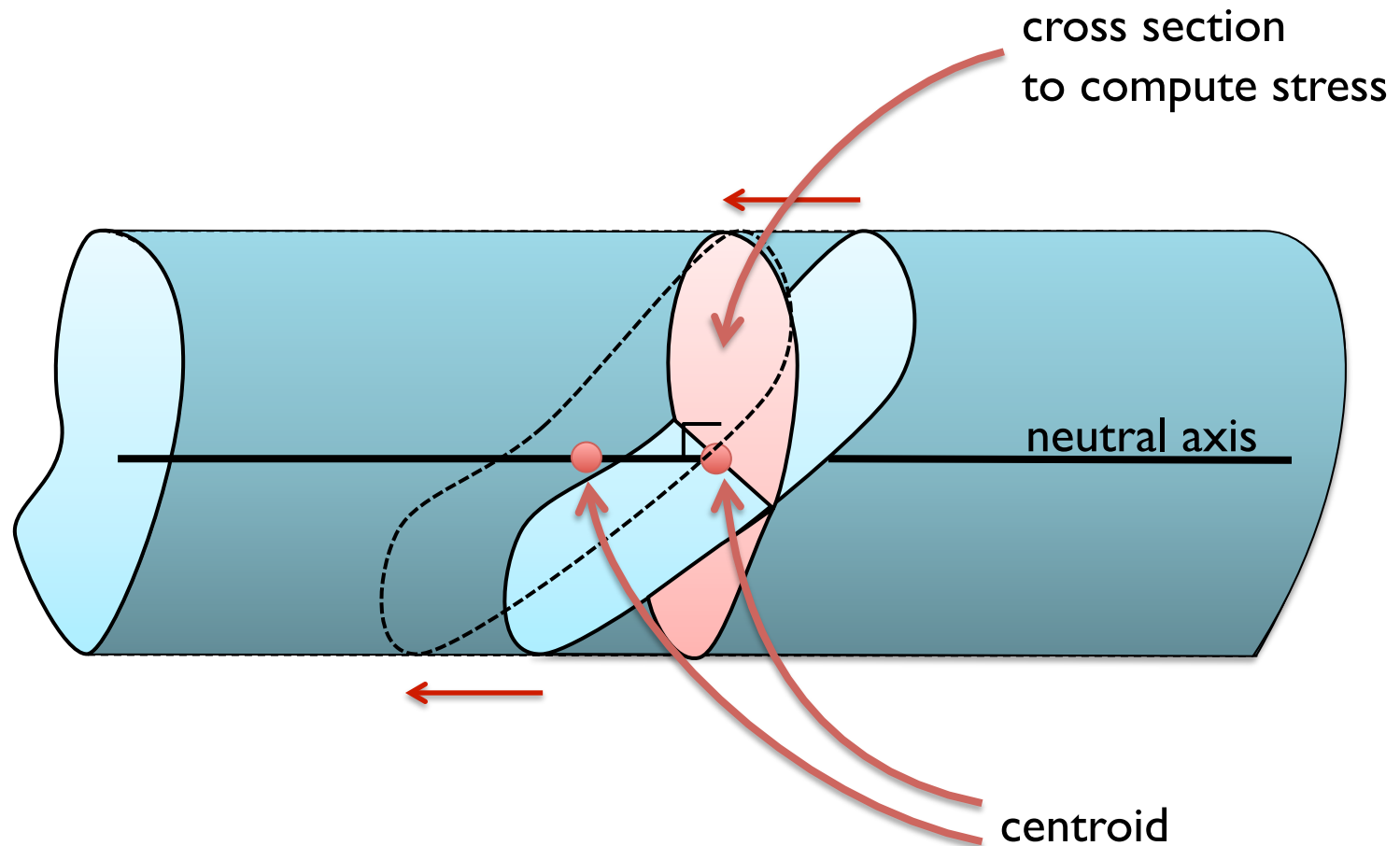
- Topology analysis of cross sections



# Neutral Axis Detection

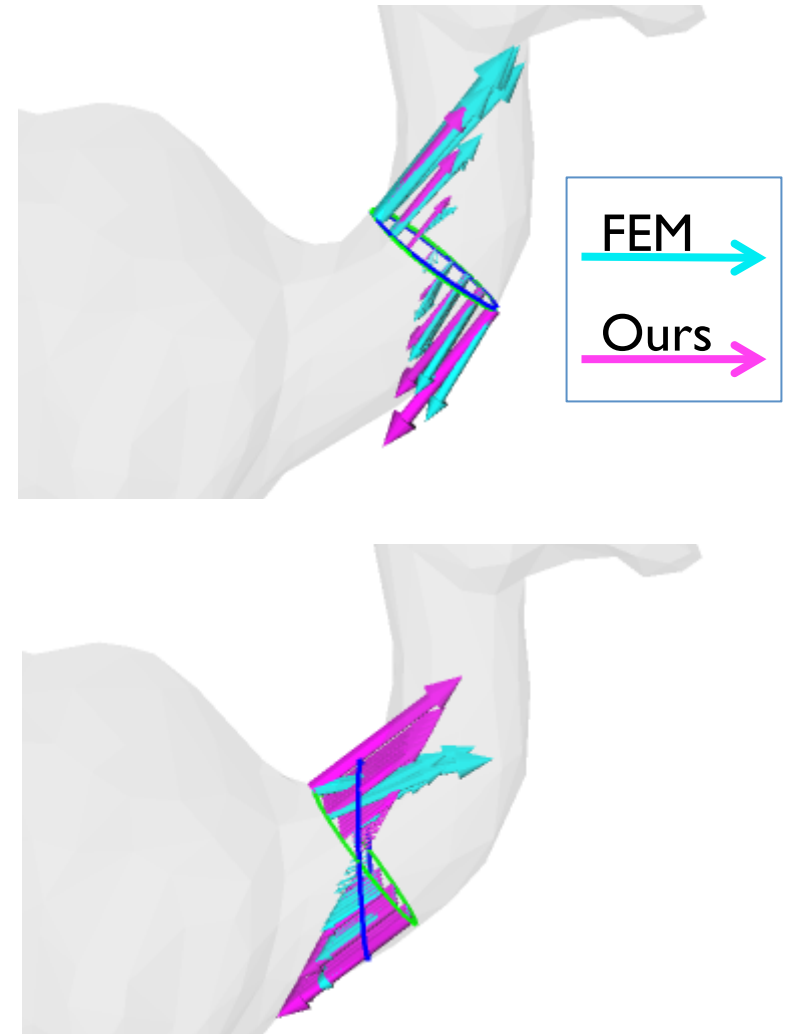
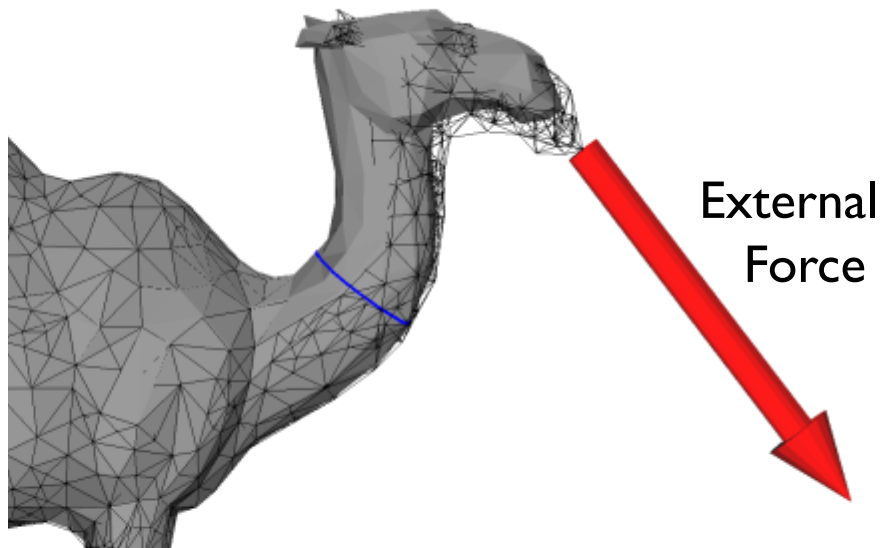
---

- Stress computation from arbitrary cross section

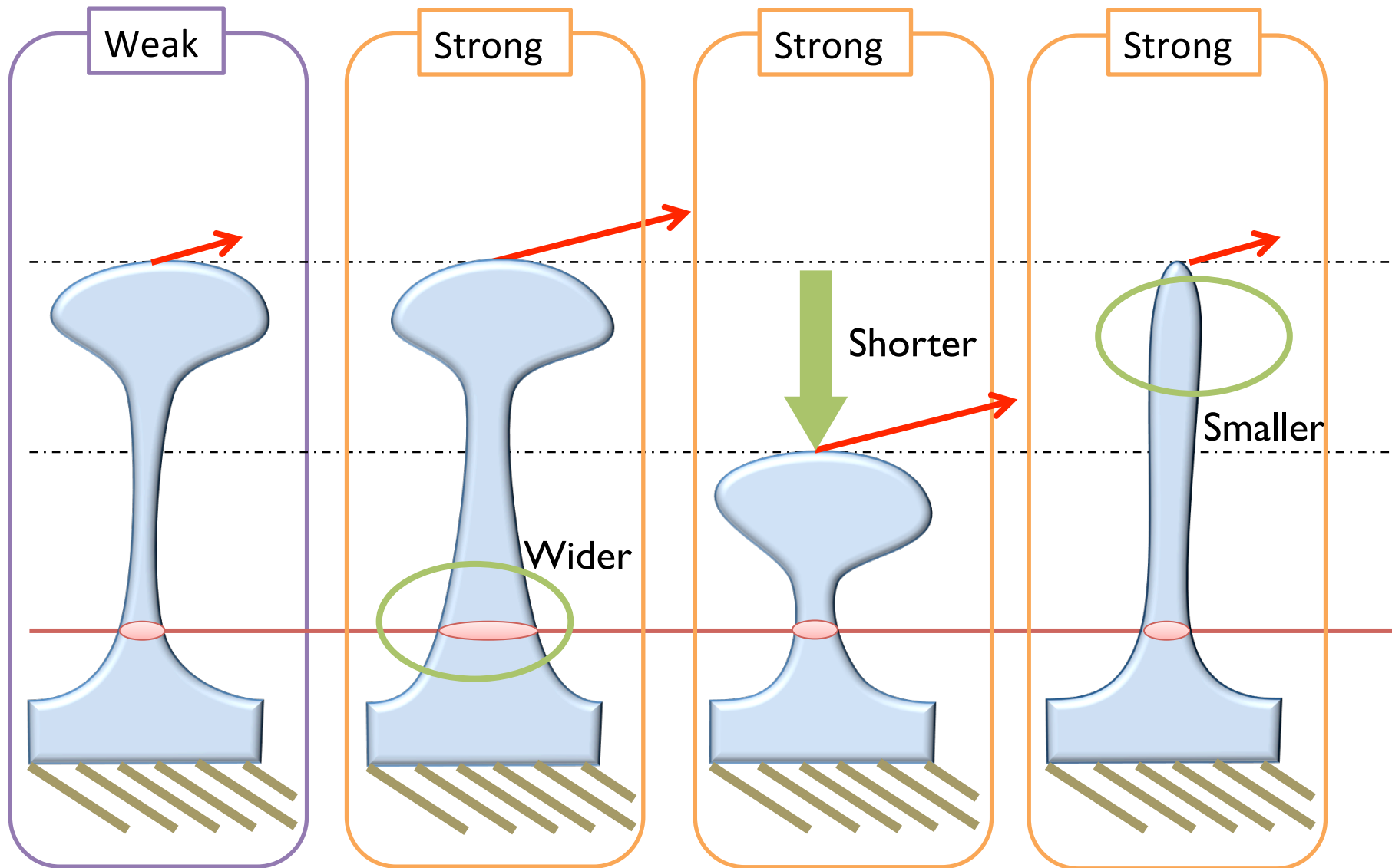


# Comparison with FEM

Linear solid material analysis  
with 2<sup>nd</sup>-order finite element method



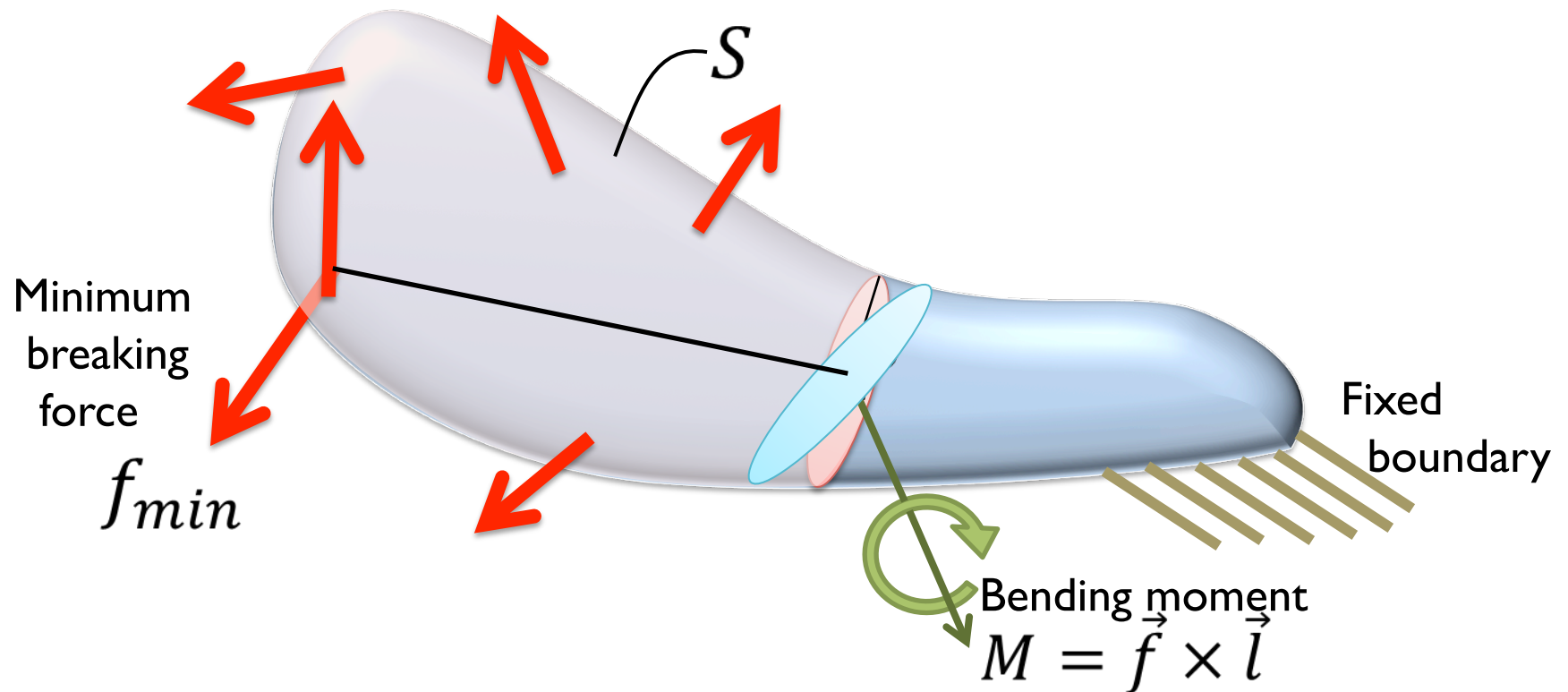
# Structural Weakness is Both Local and Global





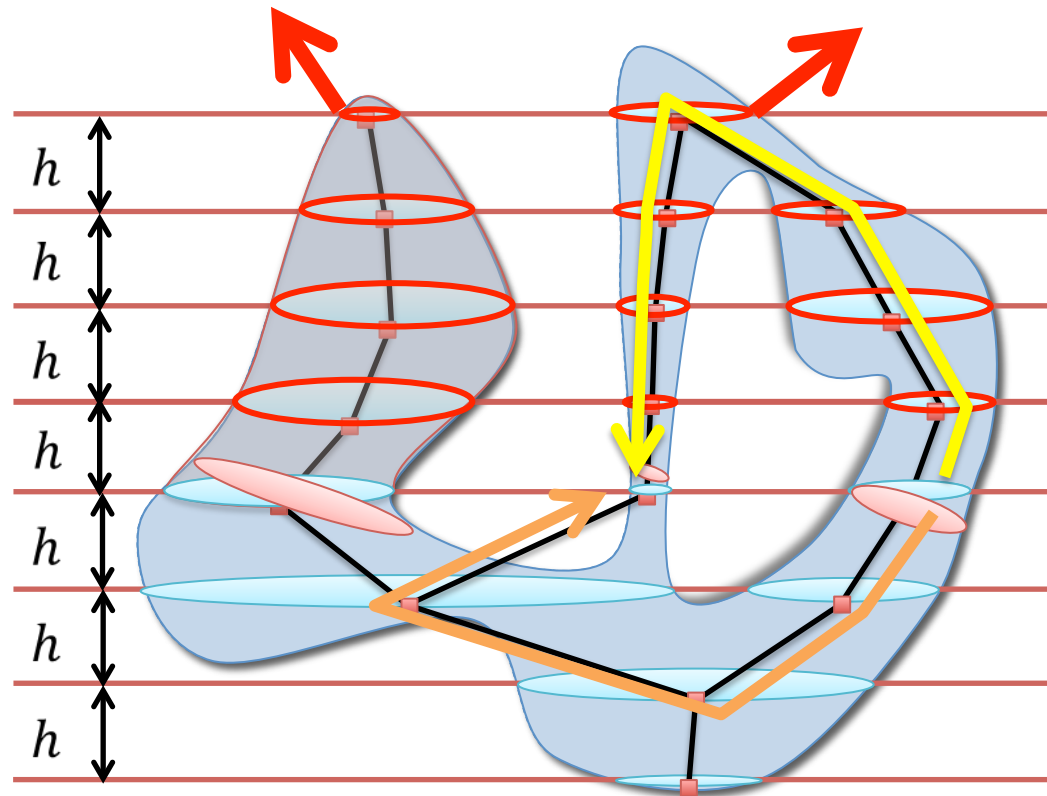
# Weakness of a Cross-Section

$$weakness = \int_S \int_{2\pi} \frac{1}{f_{min}} d\theta ds$$



# Acceleration of Weakness Computation

---

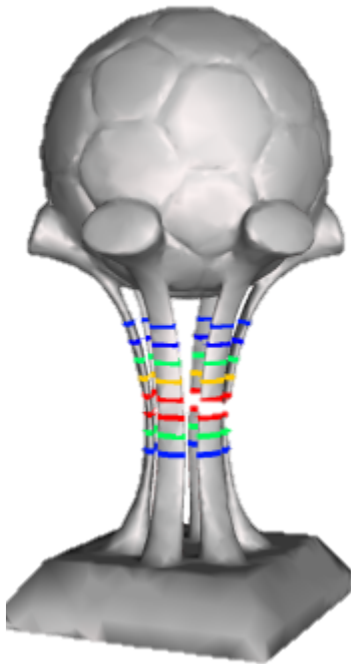


# Weak Cross Sections Performance

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- ▶ 13 directions of cross section, 32 slices in each direction

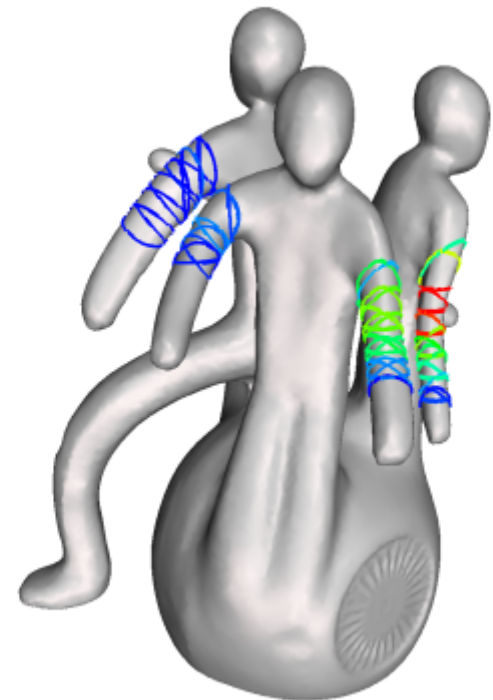
$|T| = 17k$   
n-sample=60k  
time = 0.18sec



$|T| = 34k$   
n-sample=200k  
time = 0.31sec



$|T| = 600k$   
n-sample=414k  
time = 3.8sec



**Live Demo with**

---

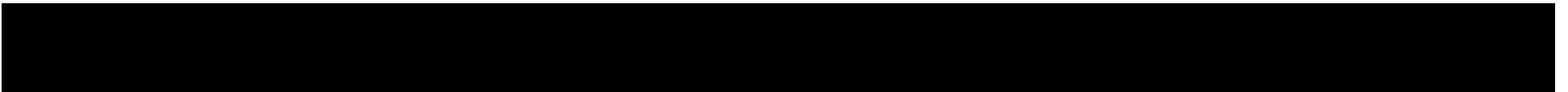


Please download and check it out !

<http://www.meshmixer.com/>

# **Application: 3D printing optimization**

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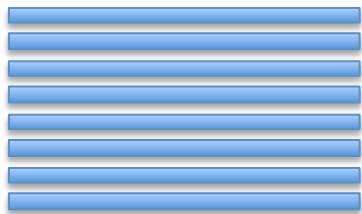


# Printed Objects Have Large An-isotropy

---



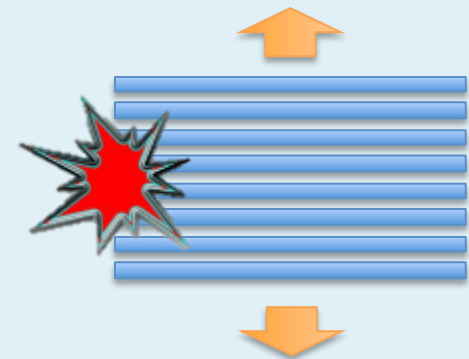
 printing  
direction



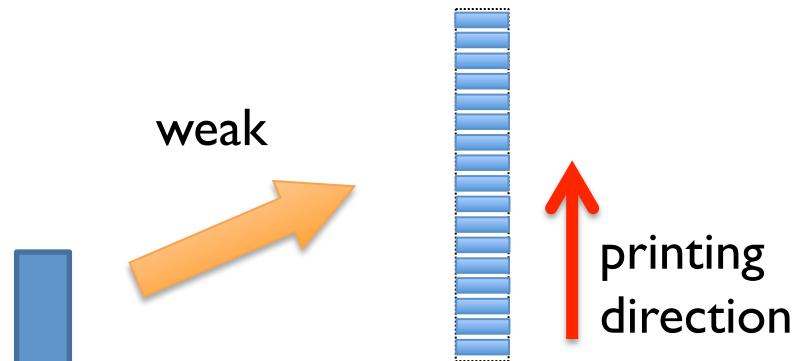
strong



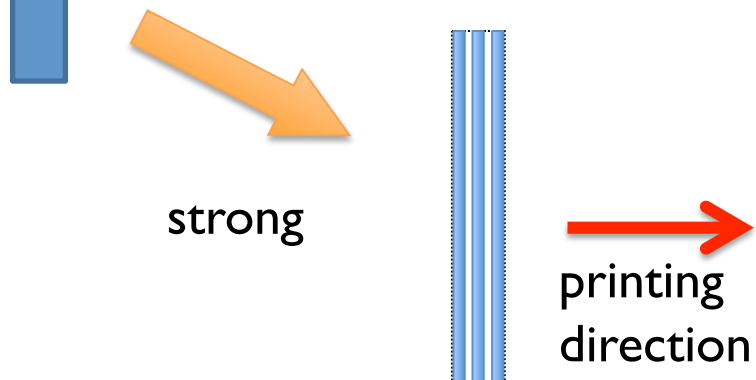
weak



# Printing Direction Determines Strength



max load: 0.22kg



max load: 3.51kg

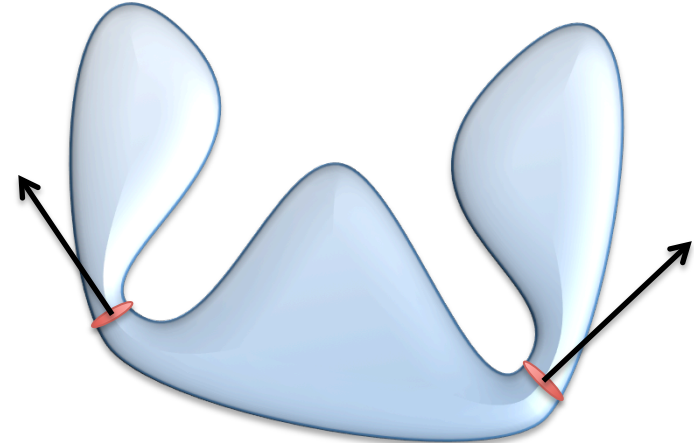
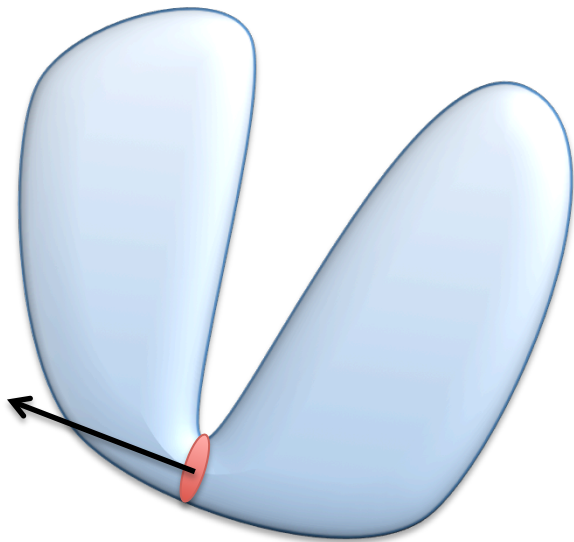
# Printing Optimization

---

- ▶ Filament should go through weak cross section



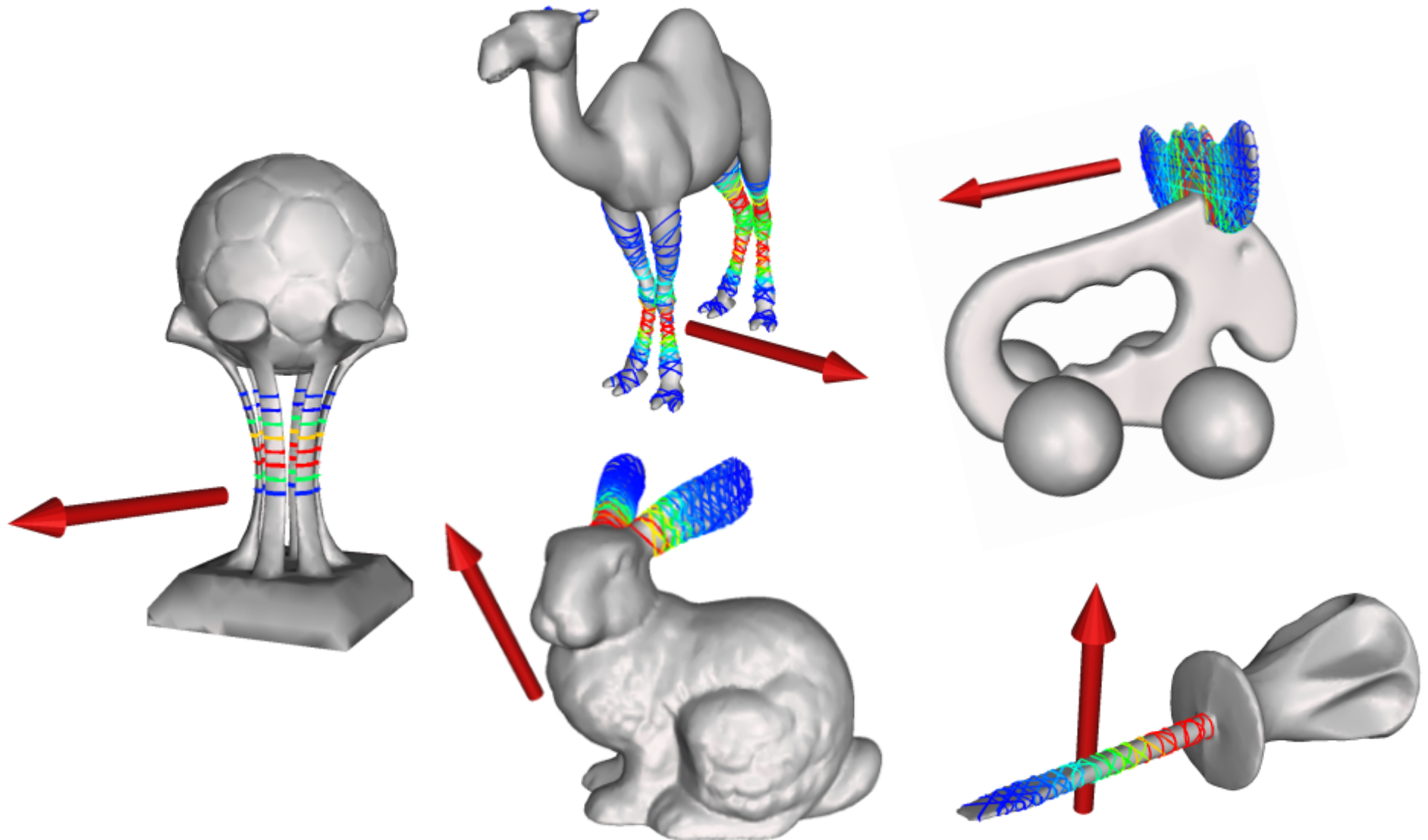
- ▶ Printing direction should be **perpendicular** to weak cross sections



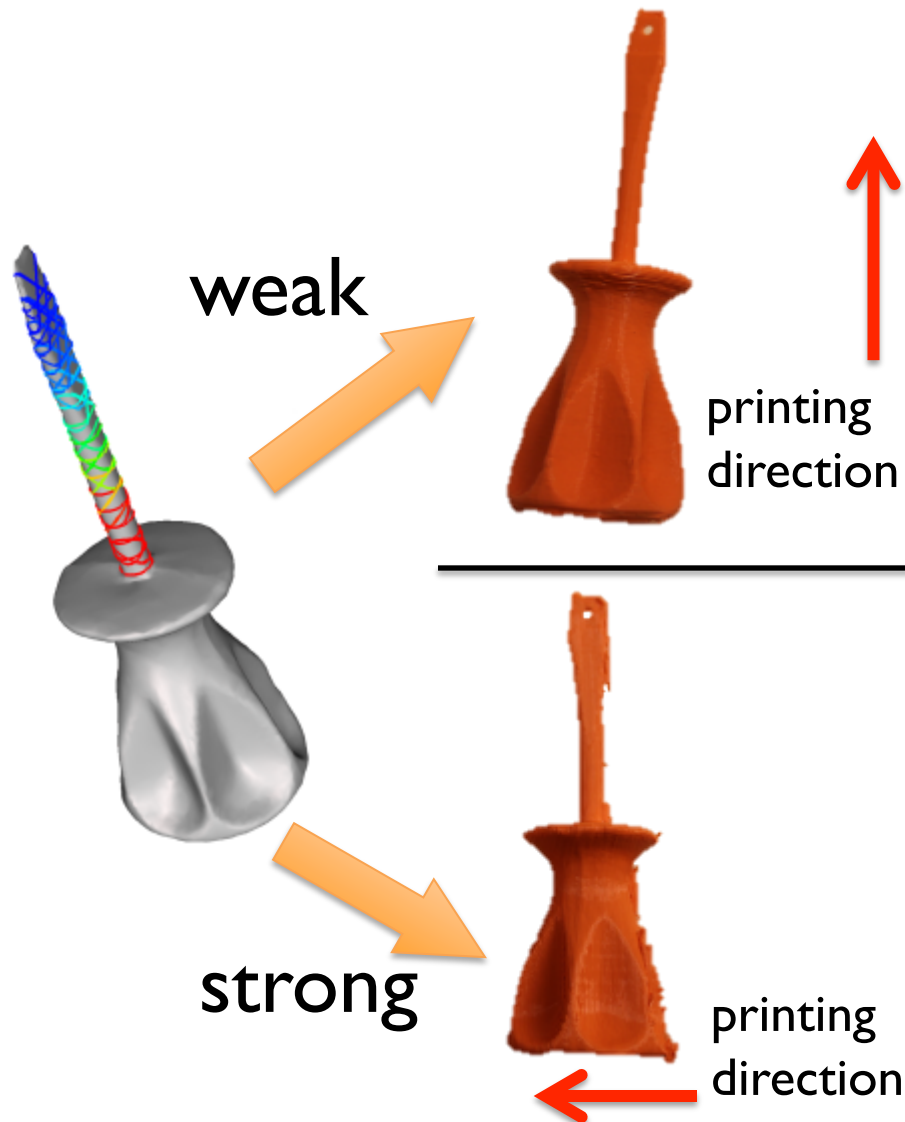


# Results

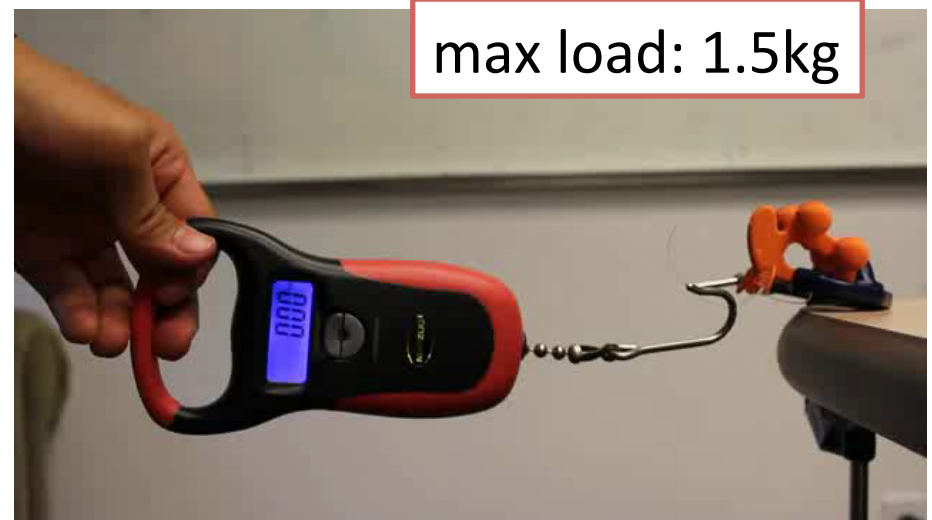
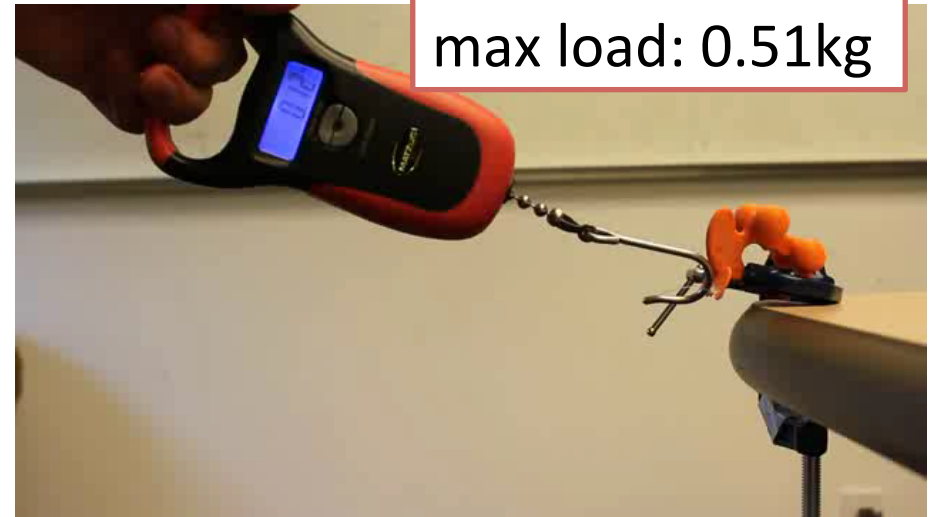
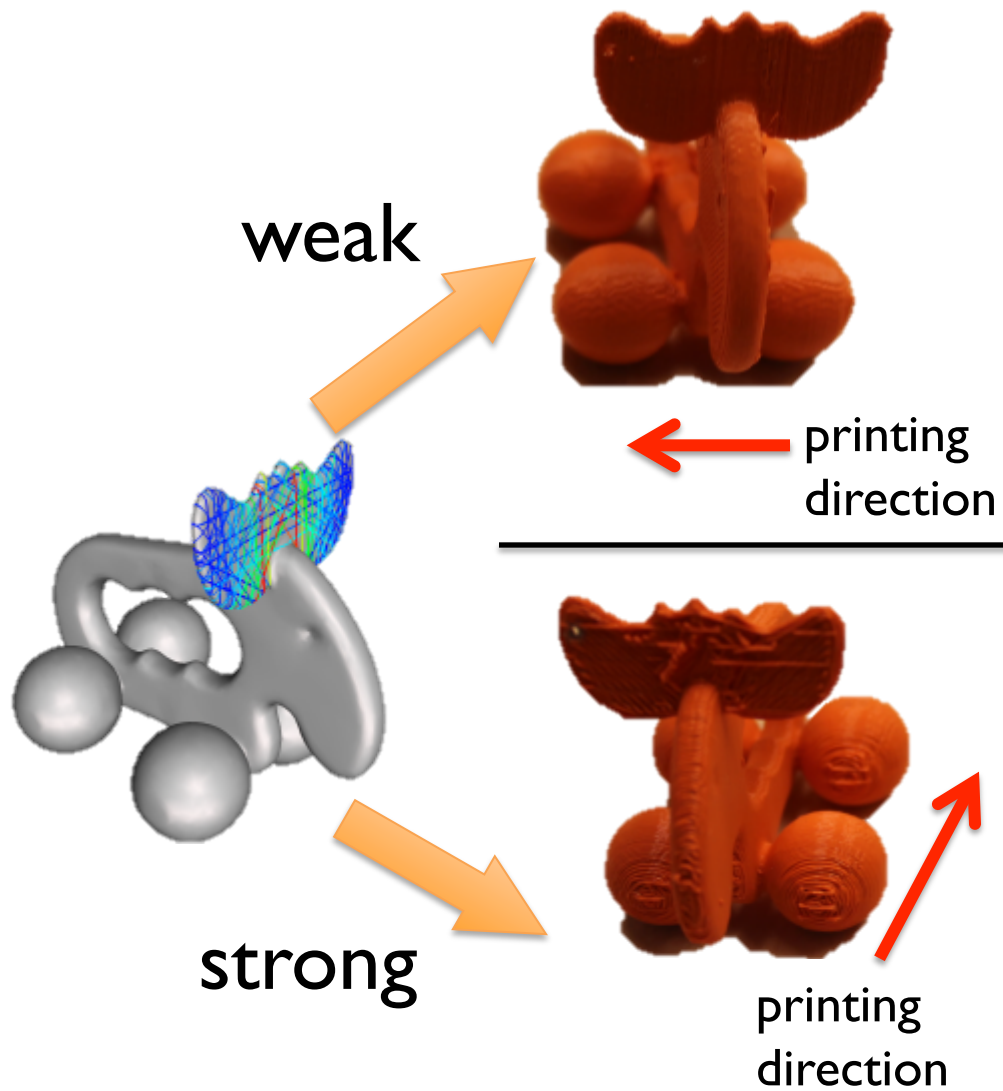
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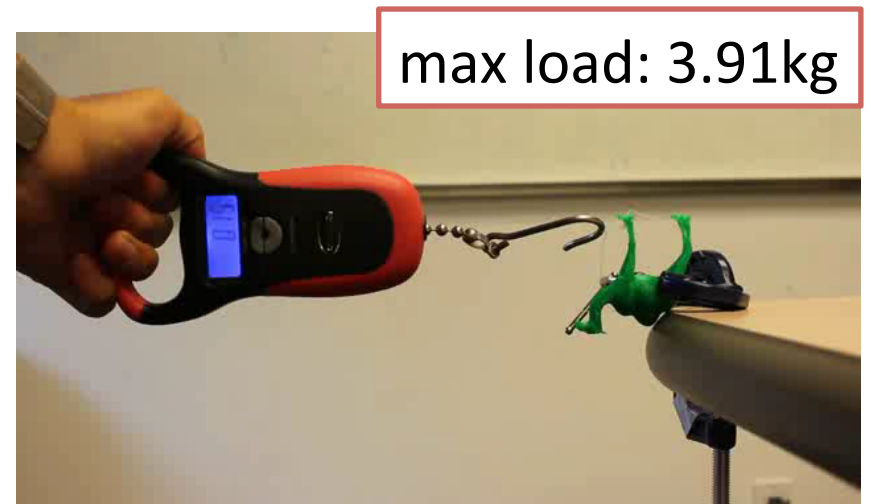
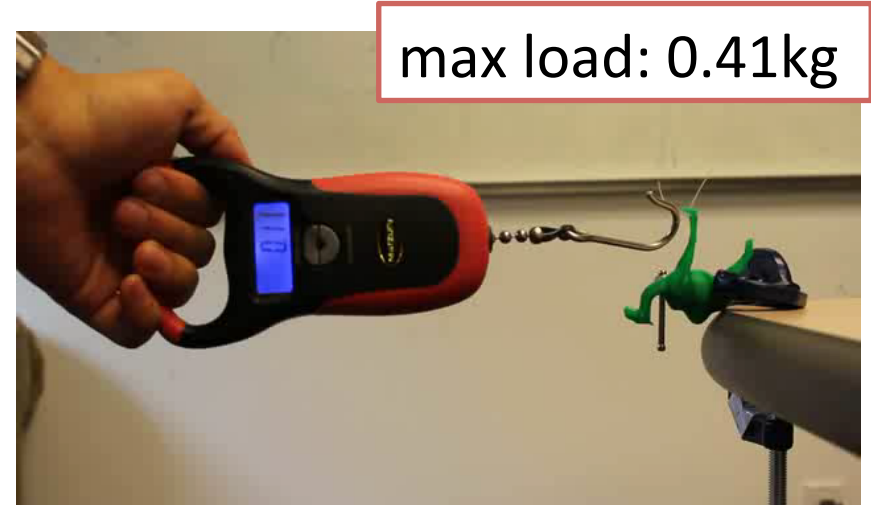
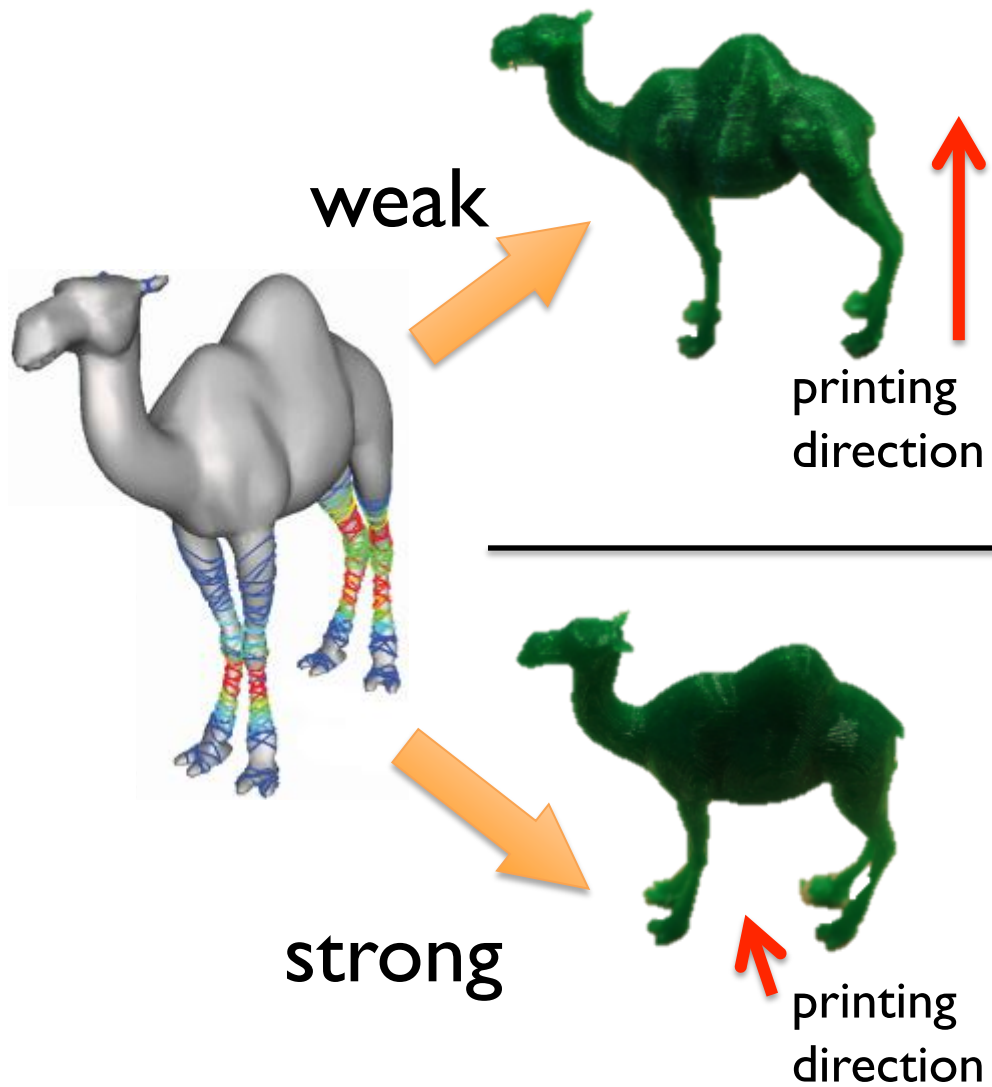
# Experiment #1



## Experiment #2

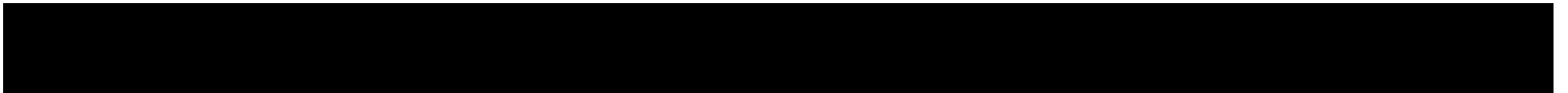


## Experiment #2



# Discussion

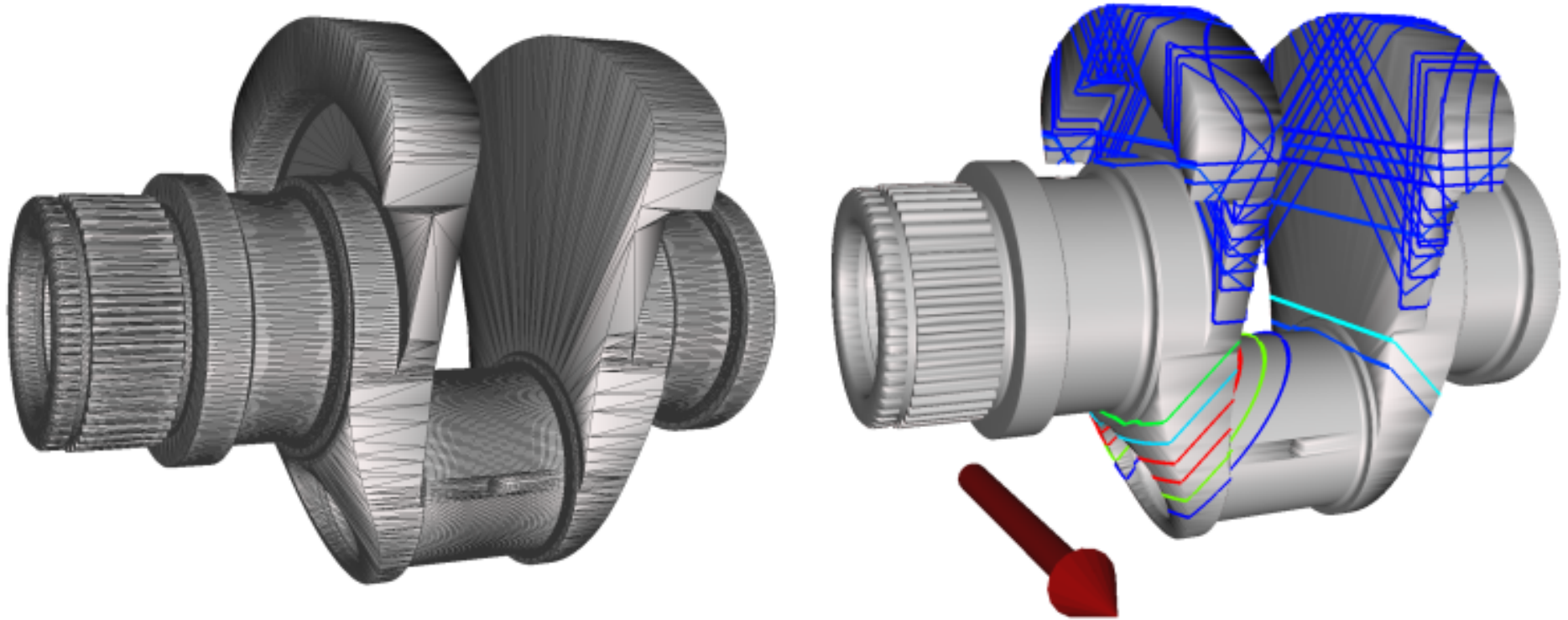
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# Analysis on Skinny Triangle Mesh

---

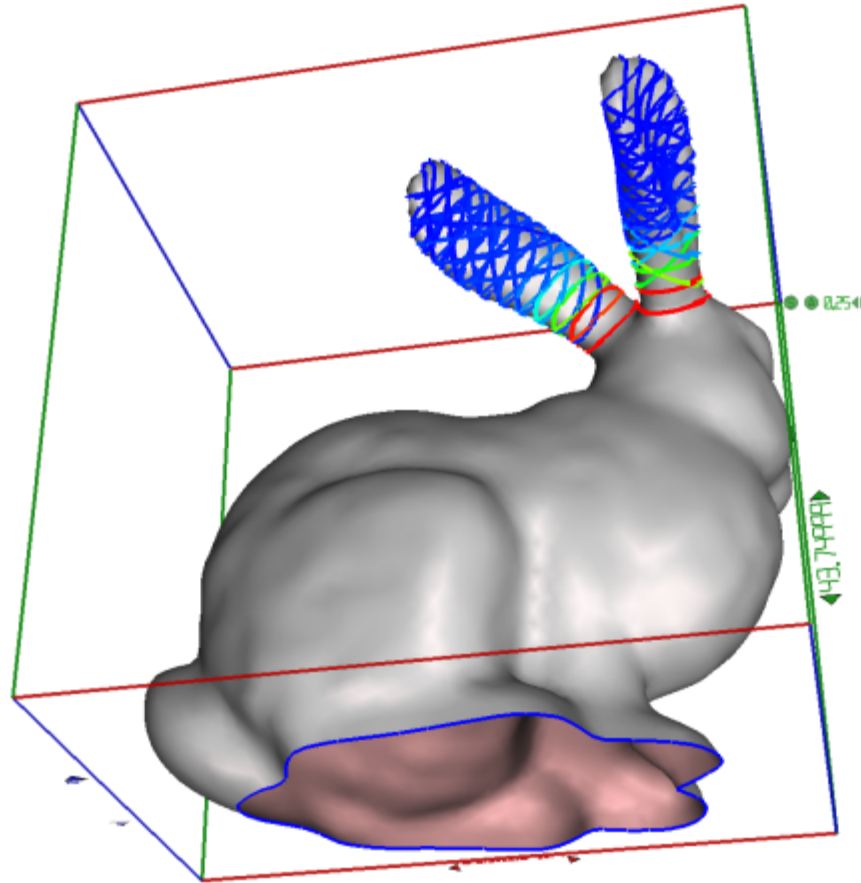
- Desirable for industrial design



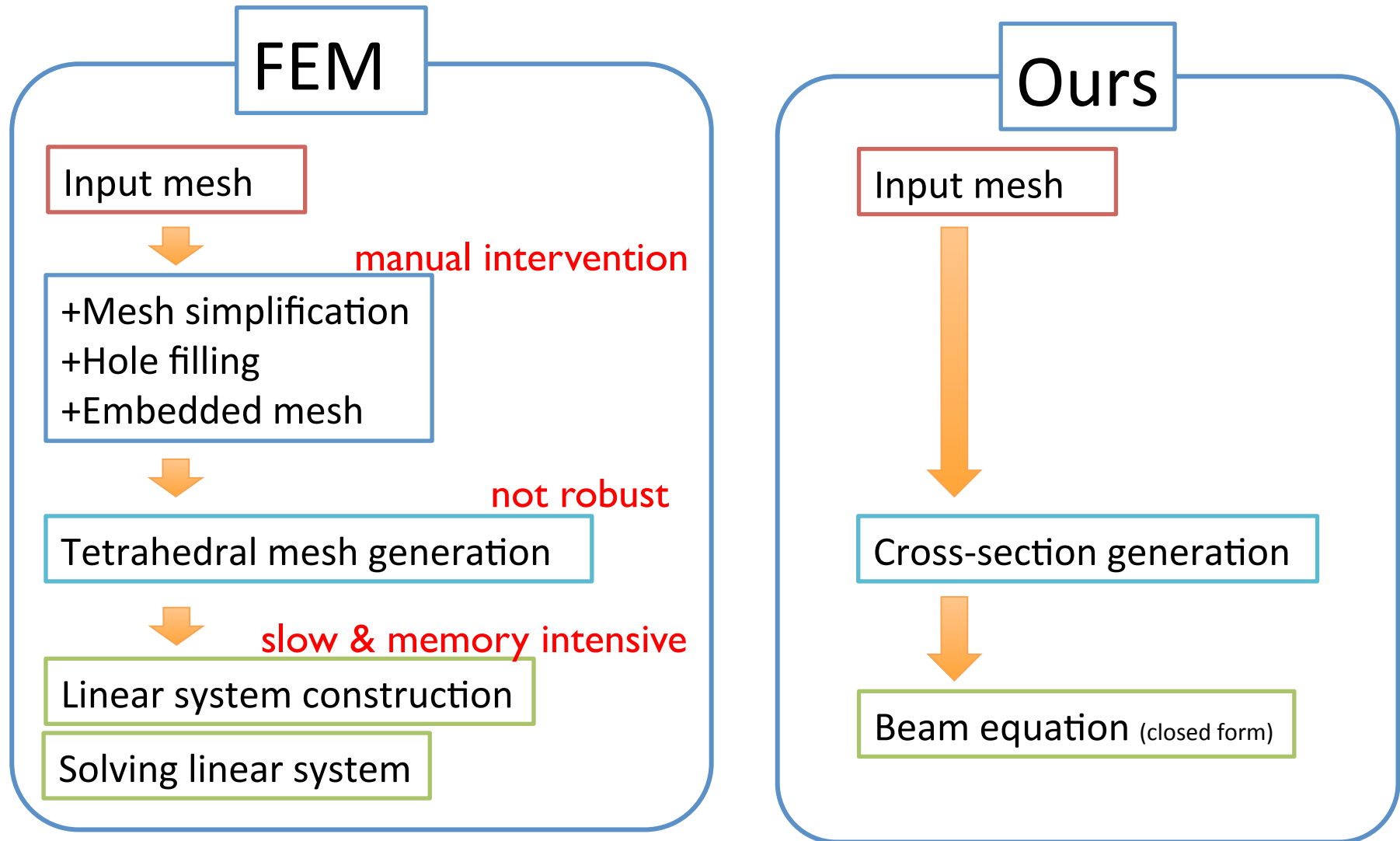
# Analysis on Holey Mesh

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- Skip the cross sections on the hole



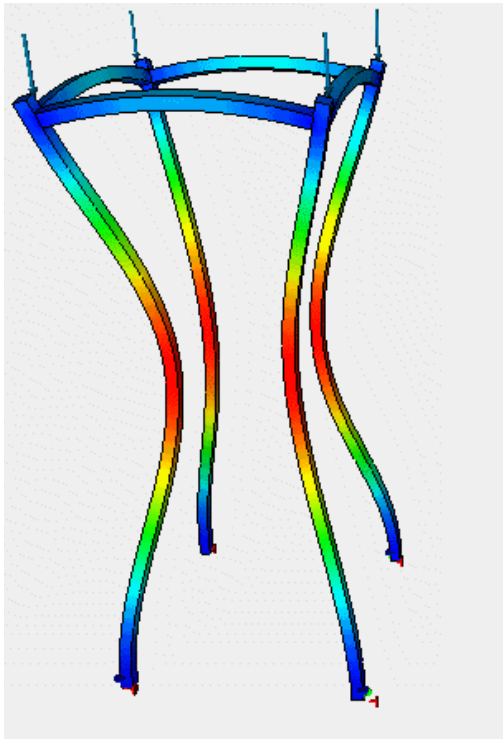
# Comparison with a Typical Procedure of FEM



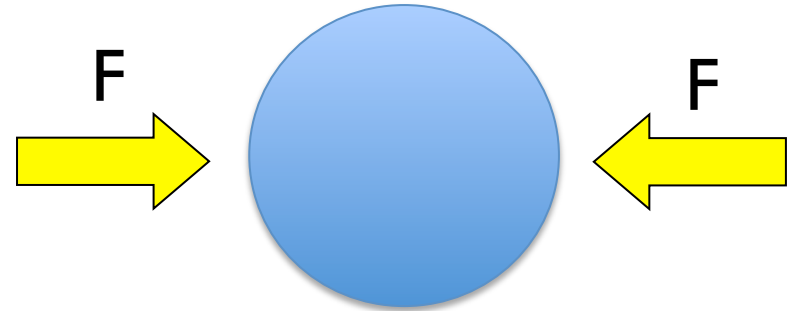


# Limitations

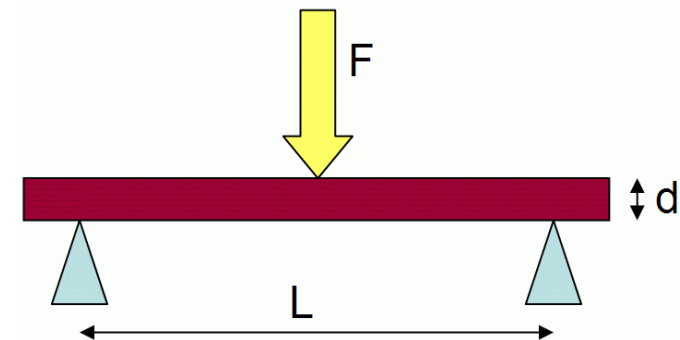
- ▶ Complex fracture is difficult  
(e.g. Buckling)



- ▶ Round object is difficult

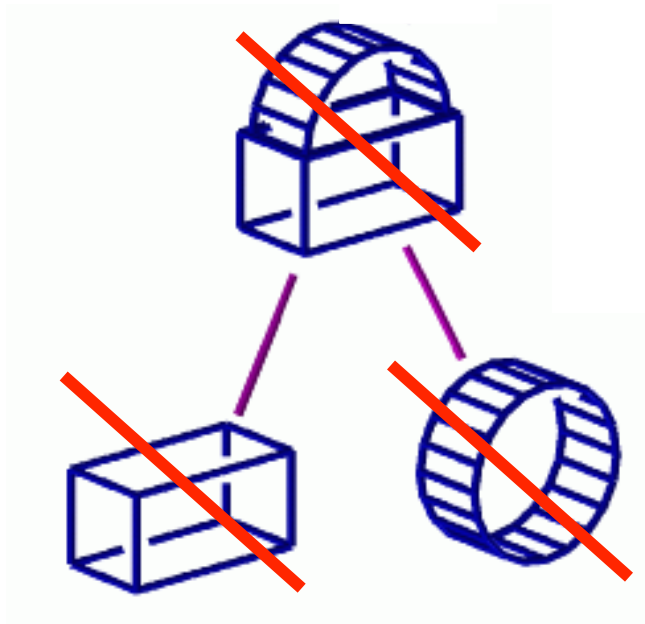


- ▶ Some boundary conditions are hard



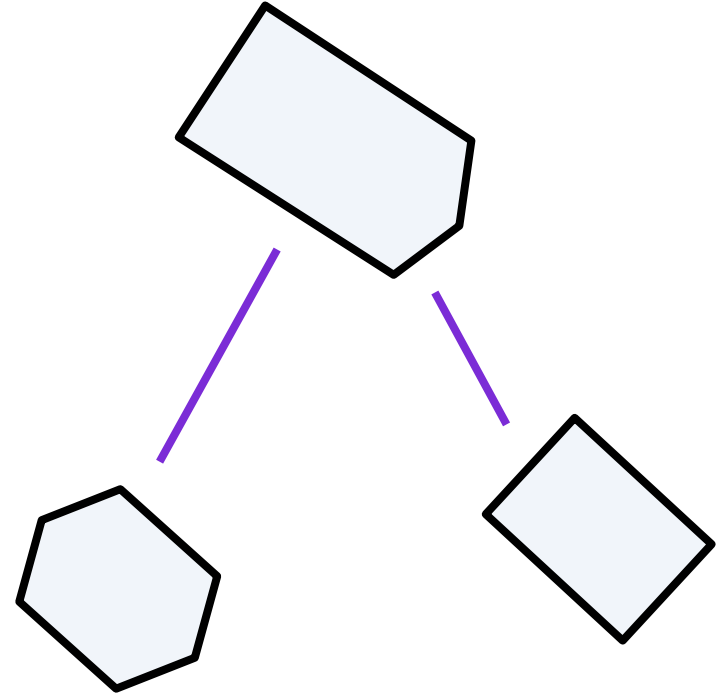
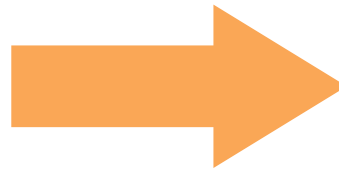
# Future Work: Direct Stress Analysis on CSG

- Real-time stress analysis for interactive CAD system



[source: IRIT user's manual]

Cross section



# Cross-sectional Structural Analysis for 3D Printing Optimization

Nobuyuki Umetani    Ryan Schmidt

